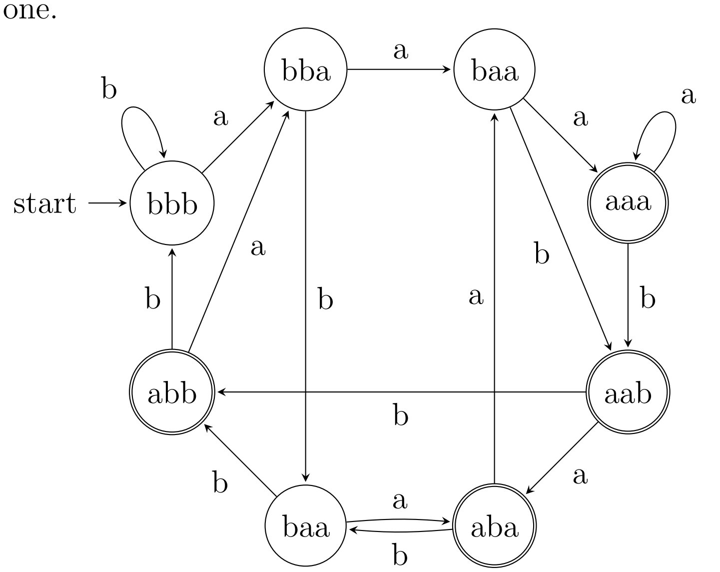
1. (30 points) The alphabet is $\{a, b\}$. Let $n \geq 0$ and let

$$L_n = \{a, b\}^* a \{a, b\}^n$$

(so the $(n+1)$th letter from the end is a).

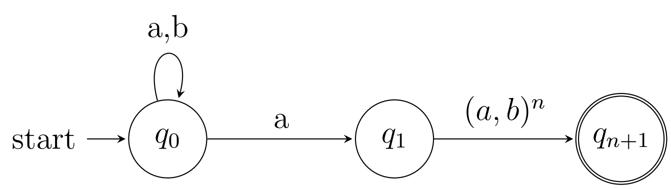
(a) (15 points) Draw a DFA for L_n when $n = 2$. Describe the DFA for L_n for any general n. How many states does L_n have in general as a function of n?

This DFA has 8 states; a DFA for L_n has 2^{n+1} states, one for each possible string a, b^n, cycling out the oldest character for the new one.



(b) (15 points) Draw an NFA for L_n for any general n. You may use DOT DOT DOT and other shortcuts. How many states does it have as a function of n?

Such an NFA has $n + 2$ states:



(c) (0 points) THINK ABOUT proving that any DFA for L_n has LOTS of states.

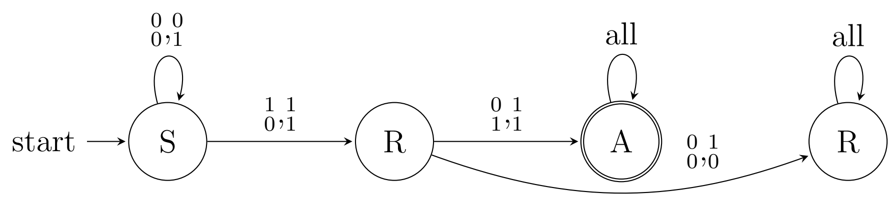
2. (30 points) Use the conventions about representing numbers and sets established in class. Your DFA's should have ACCEPT states (labelled A), REJECT states (labelled R), and STUPID states (labelled S).

   (a) (15 points) Draw a DFA for

$$\{(x, A) \mid x + 1 \in A\}$$

   How many states does it have?

   The $x + 1 \in A$ DFA has 4 states:



   (b) (15 points) For all n draw a DFA (you may use DOT DOT DOT) for

$$L_n = \{(x, A) \mid x + n \in A\}$$

   How many states does it have as a function of n?

   The $x + n \in A$ DFA has $n + 3$ states (note: all^{n-1} indicates $n - 1$ transitions for the sake of counting to n):