Homework 4 Morally Due Feb 26
THIS HOMEWORK IS TWO PAGES LONG!!!!!!!!!!!!!!!

1. (40 points) Recall that a B-NFA is an NFA where we say that an INFINITE string is accepted if there is SOME way to process it where it hits a final state infinitely often. Give an algorithm for the following: given a B-NFA M, determine if there exists an infinite string that it accepts.

   SOLUTION TO PROBLEM ONE

   We just sketch this.

   (a) Input $M = (Q, \Sigma, \delta, s, F)$

   (b) For all $f \in F$ determine: (1) is there a path from s to f (can be all e), and (2) is there a path from f back to f (can't be all e).

   (c) If there is some f such that the answer to (1) and (2) is YES then output YES. If not then output NO.

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2. (30 points) The alphabet is $\{a, b\}$. Give a B-NFA for the following languages

   In this problem note that $\{a, b\}^\omega$ means the set of INFINITE strings of a's and b's. The superscript is an ω, not a w.

   (a) (15 points)

   $$\{w \in \{a, b\}^\omega \mid w \text{ has an infinite number of } a\text{'s }\}$$

   (b) (15 points)

   $$\{w \in \{a, b\}^\omega \mid w \text{ has a finite number of } a\text{'s }\}$$

   (c) (0 points) Think about: For the above languages ponder if they could be done by a B-DFA which is a DFA where we say an infinite string accepts if it hits some final state infinitely often.

SOLUTION TO PROBLEM TWO

Omitted. REMIND ME TO DO IN CLASS

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3. (30 points) The alphabet is $\{a, b\}$. Recall that $n_a(w)$ is the number of a's in w.

   (a) (10 points) Give a regular expression for

$$\{w \mid n_a(w) \equiv 0 \pmod 3\}$$

   (b) (10 points) Give a regular expression for

$$\{w \mid n_a(w) \equiv 1 \pmod 3\}$$

   (c) (10 points) For all x, y with $0 < x < y$, give a regular expression for

$$\{w \mid n_a(w) \equiv x \pmod y\}$$

SOLUTION TO PROBLEM THREE

a)

$$b^*(b^*ab^*ab^*ab^*)^*$$

b)

$$b^*ab^*(b^*ab^*ab^*ab^*)^*$$

c) For each w, let α_w be $b^*ab^*a \cdots b^*ab^*$ where there are w a's. Then the solution is

$$\alpha_x(\alpha_y)^*$$