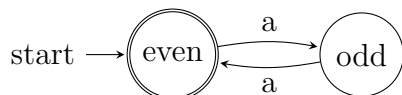2. (40 points) For each of the following state if it's REGULAR or NOT REGULAR. If it's REGULAR then give a DFA or REGEX for it. If it's NOT REGULAR then prove that.

Recall that $n_a(w)$ is the Number of a's in w. Also, recall that N (natural numbers) denotes the set of *nonnegative* integers.

(a) (8 points) (Alphabet is $\{a\}$.)
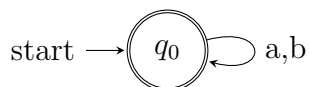
$$\{a^n a^n \mid n \in \mathsf{N}\}$$

Regular:

(b) (8 points) (Alphabet is $\{a, b\}$.) Here, x^R denotes the reverse of a string (so $(aab)^R = baa$).

$$\{xyx^R \mid x, y \in \{a, b\}^*\}$$

Regular (x can be the empty string, and thus any y is accepted):

(c) (8 points) (Alphabet is $\{a\}$.)

$$\{a^{\lceil \log_2(n+1) \rceil} \mid n \in \mathsf{N}\}$$

Regular (all natural numbers can be expressed as $\lceil \log_2(n+1) \rceil$):