

HW 6 CMSC 452. Morally Due April 9
THIS HW IS TWO PAGES LONG!!!!!!!!!!

1. (25 points) Assume $L_1 \in DTIME(T_1(n))$ and $L_2 \in DTIME(T_2(n))$. Show that $L_1 \cup L_2 \in DTIME(T_1(n) + T_2(n))$. (You can write pseudo code and note how long the program runs. We ignore multiplicative and additive constants.)
2. (25 points) Formally (using tuple notation) define a 2-dimensional Turing machine with a single halt state. Its input will be an rectangle of symbols. Also, briefly describe the transition function.
3. (25 points) Let $L \in DTIME(T(n))$. Find polynomials p such that $L^* \in DTIME(p(n)T(n))$. Give an algorithm that achieves this (it can use the algorithm for $L \in DTIME(T(n))$ and should be in pseudocode).

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4. A formula is in *DNF-form* if it is of the form

$$D_1 \vee D_2 \vee \cdots \vee D_L$$

where each D_i is a \wedge of literals. (DNF stands for Disjunctive Normal Form.) We call the d 's DISJUNCTS.

- (a) (10 points) Show that the following problem is in P: given a formula in DNF form, determine if it is satisfiable.
- (b) (8 points) Write the following CNF formula in DNF form:

$$\phi_3 = (x_1 \vee y_1) \wedge (x_2 \vee y_2) \wedge (x_3 \vee y_3)$$

How many disjuncts are in your formula?

- (c) (7 points) Write the following CNF formula in DNF form (you can describe how you would do it, but be clear).

$$\phi_n = (x_1 \vee y_1) \wedge (x_2 \vee y_2) \wedge \cdots \wedge (x_n \vee y_n)$$

How many disjuncts are in your formula?

- (d) (0 points but think about, DO NOT hand anything in for this part) Your answer to (c) should be a large function, NOT a polynomial. This means that YOUR attempt to get this CNF formula into a DNF formula causes a blowup in size. I will ask the class to vote for either
- There is a poly-sized DNF formula for ϕ_n AND this is known.
 - There is NO poly-sized DNF formula for ϕ_n AND this is known.
 - Whether or not there is a poly-sized DNF formula for ϕ_n is UNKNOWN TO SCIENCE!