HW 7 CMSC 452. Morally Due April 16
THIS HW IS TWO PAGES LONG!!!!!!!!!

1. (30 points) Let

$$IS = \{(G, k) \mid \text{ graph } G \text{ has an independent set of size } k \}$$

   (a) (10 points) Show that $CNF\text{-}SAT \leq IS$. Do not use CLIQUE as an intermediary. Explain why your reduction works.

   (b) (10 points) On the formula:

$$(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2) \wedge \neg x_3$$

   what (G, k) does your reduction produce? Draw the graph.

   (c) (10 points) On the formula:

$$(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2) \wedge x_4$$

   what (G, k) does your reduction produce? Draw the graph.

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2. (35 points) Let

$$\text{CLIQ} = \{(G, k) \mid \text{ graph } G \text{ has a clique of size } k \}$$

Let

SCLIQ be the FUNCTION that will, on input G, output the SIZE of the largest clique.

Let

FINDCLIQ be the FUNCTION that will, on input G, output both the SIZE of the largest clique, and SOME clique of that size (that is, a list of vertices that form a clique of max size).

(a) (17 points) Show that if CLIQ $\in P$ then SCLIQ can be computed in polynomial time. (ADDED LATER TO CLARIFY: The *CLIQ TM* is the Poly Time TM that decides the set CLIQ.) (THINK ABOUT but don't hand in: Your algorithm made several calls to the CLIQ TM. How many? Assuming $P \neq NP$, is there a way to do with this with 18 calls to CLIQ?)

(b) (18 points) Show that if CLIQ $\in P$ then FINDCLIQ can be computed in polynomial time. (THINK ABOUT but don't hand in: Your algorithm made several calls to the CLIQ TM. How many? Assuming $P \neq NP$, is there a way to do with this with 18 calls to CLIQ?)

3. (35 points) Let

$$3COL = \{G : \text{ graph } G \text{ is 3-colorable}\}.$$

Show that $3COL \leq SAT$. Give an explicit reduction and explain why it works.

(HINT: Let G have vertices $1, \ldots, n$. The variables are, for $1 \leq i \leq 3$, $1 \leq j \leq n$, x_{ij}. The variable x_{ij} is intended to be TRUE if Vertex j is colored i, and FALSE if not.)