HW 8 CMSC 452. Morally Due April 23
THIS HW IS TWO PAGES LONG!!!!!!!!!

1. (30 points) A *poly inequality* is an inequality of the form

$$p(x_1, x_2, \ldots, x_n) \leq c$$

where $p(x_1, \ldots, x_n)$ is a polynomial with integer coefficients WITHOUT a constant term, and $c \in \mathsf{Z}$.

TWO EXAMPLES:

$$x_1^3 x_4^2 - 2x_2 x_3 + 18x_3^{14} x_4^2 + x_1 \leq 1000.$$

$$x_1 + x_2 \leq 89$$

Let POLY PROGRAMMING, called PP, be the following problem:

Given a set of poly inequalities determine if there is some way to set the variables to rationals so that all the inequalities hold.

(a) Show that 3-$SAT \leq PP$.

(b) Use your reduction on the following formula (i.e., list the inequalities produced by the reduction)

$$(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4) \wedge (x_2 \vee \neg x_3 \vee \neg x_4)$$

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2. (40 points) Let

$$\text{CLIQ17} = \{G \mid \text{ graph } G \text{ has a clique of size 17 } \}$$

   (a) (25 points) Either show that CLIQ17 is in P or show that CLIQ17 is NP-complete or do both. (ALSO — not to hand in, but think about — is it likely that someone in the class will be able to do both?)

   (b) (25 points) Is CLIQ17 closed under minors (see Wikipedia entry for clarification). That is, if $G \in$ CLIQ17 and H is a minor of G, is it necessarily true that $H \in$ CLIQ17? If so then prove it, if not then give a counterexample.

   `https://en.wikipedia.org/wiki/Graph_minor`

3. (30 points) Let

$$FACT = \{(n, x) \mid \text{ there is a nontrivial factor of } n \text{ that is } \leq x \ \}.$$

   (A NONTRIVIAL factor of n is a positive factor that is NOT 1 and NOT n.)

   n and x are both positive integers and are given in binary, so the NUMBER (say, for example) ONE THOUSAND only takes around 10 bits, NOT 1000 bits, to input.

   Let $FFACT$ be the function that, on input n, outputs the complete prime factorization of n.

   Show that if $FACT \in P$ then $FFACT$ can be computed in Polynomial time.

   NOTE- poly in the LENGTH of the input. So the LENGTH of ONE THOUSAND would be TEN. So $FACT \in P$ means that it takes time $p(\log n + \log x)$ to decide (n, x) for some poly p.