OPTIONAL PROJECT TO GET YOU FROM F to D or D to $C-$
THIS DOCUMENT IS TWO PAGES LONG
Morally Due May 14, 12:30PM, Dead-Cat May 16 12:30PM

If you do this project AND end up with an F or D in the course then I will grade it and MAY use your grade to BUMP your grade up (from an F to a D, from a D to a C-). Throughout this document "prove" means "give a construction and discuss why it works." What you hand in must be TYPED or written with VERY GOOD HANDWRITING.

DUE the LAST DAY of class. Absolute Deadline.

HINT: START early. Feel free to get help from me or the TA.

CONVENTION: You can't say 'by theorem BLAH'. For example, if I want you to show that

If L is regular than LL is regular

you CANNOT say

because regular langs are closed under concatenation

1. (0 points but you have to answer) What is your name? Write it clearly.

2. Assume the alphabet is $\{a, b\}$. Let \$ be another symbol. Let L_1 and L_2 be regular subsets of $\{a, b\}^*$. Show that

$$\{x\$y \mid x \in \{a, b\}^* - L_1 \text{ and } y \in L_2\}$$

   is regular.

3. Let L be in P. Prove or disprove or state that it is unknown to science.

   (a) L^R is in P. (Recall that L^R is all strings in L written backwards, so if $aab \in L$ then $baa \in L^R$.)

   (b) L^* is in P.

4. Let L be in NP. Prove or disprove or state that it is unknown to science.

   (a) L^R is in NP.

   (b) L^* is in NP.

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5. Let L be decidable. Prove or disprove or state that it is unknown to science.

   (a) L^R is decidable.

   (b) L^* is decidable.

6. For this problem you may assume regular languages are closed under UNION, INTERSECTION, COMPLEMENTATION, and PROJECTION. Describe carefully an algorithm that will, on input a SENTENCE ϕ in PRESBURGER ARITHMETIC, output (1) TRUE if ϕ is TRUE, and (2) FALSE if ϕ is FALSE. (Note that since ϕ is a sentence it is either true or false.) (On the last page we define Presburger formally for you. We only do a subset and that's all you will need to do.)

7. (a) Describe the reduction of SAT to IND SET. That is, describe how you would take a formula ϕ (we can assume its in CNF form) and from it get a graph G and a number k in POLYNOMIAL TIME such that

$$\phi \in SAT \text{ iff } (G,k) \in \text{IND SET}$$

   (b) Use the answer to part 1 to find a graph G and a number k such that

$$(x_1 \vee \neg x_2) \wedge (\neg x_1 \vee \neg x_3) \wedge (x_1 \vee x_2 \vee x_3) \in SAT$$

   iff

   $(G,k) \in$ IND SET.

Presburger Arithmetic

(We do not do the full Presburger as that would be too complicated.)

The variables range over N. Yes N includes 0.

The following are formulas in PA.

Atomic Formulas:

- $x + y = z$
- $x + c = z$ where $c \in \mathsf{N}$

Building up formulas: If ϕ_1 and ϕ_2 are formulas then the following are formulas:

- If $\phi_1 \wedge \phi_2$
- $\phi_1 \vee \phi_2$
- $\neg\phi_1$
- If ϕ has free variable x then $(\exists x)[\phi(x)]$ is a formula

RECALL: A sentence is a formula with no free variables. Hence it can be TRUE or FALSE.

NOTE: Full Presburger would include things like $x + y + z = w$ which we do not include in order to make your life easier.