452 MIDTERM

Do not open this exam until you are told. Read these instructions:

1. This is a closed book exam, though ONE sheet of notes is allowed. **No calculators, or other aids are allowed**. If you have a question during the exam, please raise your hand.
2. There are 4 problems which add up to 100 points. The exam is 1 hours 15 minutes.
3. For each question show all of your work and **write legibly**. **Clearly indicate** your answers. No credit for illegible answers.
4. After the last page there is paper for scratch work. If you need extra scratch paper **after** you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper **will not** be graded.
5. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*"

6. Fill in the following:

NAME :
SIGNATURE :
SID :
SECTION NUMBER :

SCORES ON PROBLEMS

Prob 1:	
Prob 2:	
Prob 3:	
Prob 4:	
TOTAL	

1. (25 points) For the next three problems fill in the blanks. No explanation required.

   (a) If there exists a DFA that recognizes L_1 with n_1 states and a DFA that recognizes L_2 with n_2 states, then there exists a DFA for $L_1 \cap L_2$ with BLANK states.

   BLANK is:

   (b) If there exists a DFA that recognizes L with n states, then there is a DFA that recognizes $\overline{L}$ (also called $\Sigma^* - L$) with BLANK states.

   BLANK is:

   (c) If there exists an NFA that recognizes L with n states, then there is a DFA that recognizes L with BLANK states.

   BLANK is:

   (d) If there exists a regular expression for L_1 of length n_1 and a regular expression for L_2 of length n_2, then there exists a regular expression for $L_1L_2 = \{xy \mid x \in L_1, y \in L_2\}$ with BLANK states.

   BLANK is:

   (e) If there exists a DFA for L over the alphabet $\{00, 01, 10, 11\}$ with n states then there exists an NFA for the projection of L on the first coordinate over the alphabet $\{0, 1\}$ with BLANK states.

   BLANK is:

SOLUTION TO PROBLEM ONE

You did not have to give an explanation but I will.

(a) $n_1 n_2$. The cross product construction.

(b) n. Swapping final and non-final states does not change the number of states.

(c) 2^n. The powerset construction.

(d) $n_1 + n_2$. This is just one one reg exp followed by another. We also accepted $n_1 + n_2 + 1$ in case you though you needed a DOT for concat.

(e) n. When you project you go from a DFA to an NFA but do not change the number of states.

END OF SOLUTION TO PROBLEM ONE

2. (25 points) We use the WS1S convention. Recall that, for example $(2, 5, \{0, 3, 5, 6\})$ is represented as follows:

$$\begin{array}{ccccccc} 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 1 & * \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}$$

where the $*$ can be either 0 or 1.

Write down DFA's for the following. Label states A (for Accept), R (for Reject), and S (for Stupid).

(a) $\{(x, y) : x \le y + 1\}$.

(b) $\{(x, y) : x \le y + 1000\}$. (For this one, you can and should use DOT DOT DOT.)

SOLUTION TO PROBLEM TWO

Omitted since hard to draw DFA's in LaTeX.

END OF SOLUTION TO PROBLEM TWO

3. (a) (10 points) We define *extended regular Expressions* as regular expressions that also: (1) allows numerical exponents so you can write things like a^{1000}, which of course means 1000 a's, and (2) allows complementation, so you can write things like $(a \cup b)^i - a^i$.

Give an extended regular expression for the set

$$L = \{w : w \text{ has } aa \text{ as a prefix but DOES NOT have } aa \text{ as a suffix } \}$$

(**Examples:**

aa is NOT in L since it has aa as a suffix.

aaa is NOT in L since it has aa as a suffix.

aab is in L since it has aa as a prefix and does not have aa as a suffix.

$aaab$ IS in L since it has aa as a prefix but does not have aa as a suffix.

)

Hint: Make sure your extended regular expression DOES include aab and $aabb$ but and DOES NOT include aa or aaa.

(b) (15 points) Give an extended regular expression for the set

$$L = \{w : w \text{ has } a^{1000} \text{ as a prefix but DOES NOT have } a^{1000} \text{ as a suffix } \}$$

(You may use DOT DOT DOT or unions such as $\bigcup_{i=1}^{948}$.)

SOLUTION TO PROBLEM THREE

1) Some people answered

$$aa(a \cup b)^*(ab \cup ba \cup bb).$$

This is not quite correct since it leaves out aab. What to do about that? The above expression is fine for strings of length ≥ 4. We must include the strings of length ≤ 3 directly.

No string of length 0 or 1 is in since can't have aa as a prefix.

No string of length 2 can be in since if it has aa as a prefix it has aa as a suffix.

The only string of length 3 that can be in is aab.

Hence one possible answer is:

$$aa(a \cup b)^*(ab \cup ba \cup bb) \cup \{aab\}.$$

There is also an alternative, slightly simpler expression. If we wanted strings that have aa as a prefix, and don't care about the suffix, we could write $aa(a \cup b)^*$. But we also want to *disallow* strings of the form $(a \cup b)^*aa$, which are those with aa as a suffix. We can use complementation to get our final answer:

$$aa(a \cup b)^* - (a \cup b)^*aa$$

2) Some people answered

$$a^{1000}(a \cup b)^*((a \cup b)^{1000} - a^{1000}).$$

This is not quite correct since it leaves out any expression of length ≤ 1999. What do do about that? The above expression is fine for strings of length ≥ 2000. We must include the strings of length ≤ 1999 directly.

No string of length 0, 1, 2,... 999 is in since can't have a^{1000} as a prefix.

No string of length 1000 can be in since if has a^{1000} as a prefix then its also a suffix.

Let $1 \leq i \leq 999$. The only strings of length $1000 + i$ are of the form $a^{1000}((a \cup b)^i - a^i)$.

So one possible answer is

$$a^{1000}(a \cup b)^*((a \cup b)^{1000} - a^{1000}) \cup \bigcup_{i=1}^{999} a^{1000}((a \cup b)^i - a^i).$$

Some version that uses DOT DOT DOT instead of the big union is fine also, so long as its clear.

As with the previous part, we can get a simpler expression:

$$a^{1000}(a \cup b)^* - (a \cup b)^* a^{1000}$$

END OF SOLUTION TO PROBLEM THREE

4. (25 points) Show that ANY DFA for

$$L = \{a^i : i \neq 1000\}$$

has to have at least 1000 states.

SOLUTION TO PROBLEM FOUR

Assume, by way of contradiction, that L has a DFA M with ≤ 999 states.

Feed the string a^{1000} into M and look at the resulting sequence of states (not counting a^0). It will be 1000 long and end in a NON-final state. Since the DFA has ≤ 999 states some state repeats. Call it q. Let s be the start state, q be the state that repeats, and $NOTF$ be the state that a^{1000} ends up in (a NOT Final state)

The sequence of states is:

$s, \ldots, q, \ldots, q, \ldots, NOTF$.

Let k be the number of a's that take you from the first q to the second q (so the number of a's in the loop). Then note that

a^{1000} ends up in state $NOTF$ and is rejected, as it should be

but

a^{1000+k} ends up in state $NOTF$ and is rejected WHICH IT SHOULDN"T BE.

END OF SOLUTION TO PROBLEM FOUR

Scratch Paper