# 1   3-SAT to CLIQ

In this project you will be exploring the reduction 3-SAT $\leq$ CLIQ. Recall that a 3-SAT formula is a Boolean formula that is a conjunction of disjunctions with three variables. Here is an example with three clauses:

$$(x_1 \lor x_2 \lor \neg x_3) \land (\neg x_1 \lor \neg x_2 \lor x_3) \land (x_1 \lor \neg x_2 \lor x_4)$$

This formula is satisfiable if and only if the following graph contains a 3-clique with exactly one vertex from each set of three vertices. Each set represents a clause. Each variable from the formula is a vertex in this graph with an edge drawn between every pair of verices except for those that contradict each other (such as x_1 and $\neg x_1$) and those from the same clause in the formula.
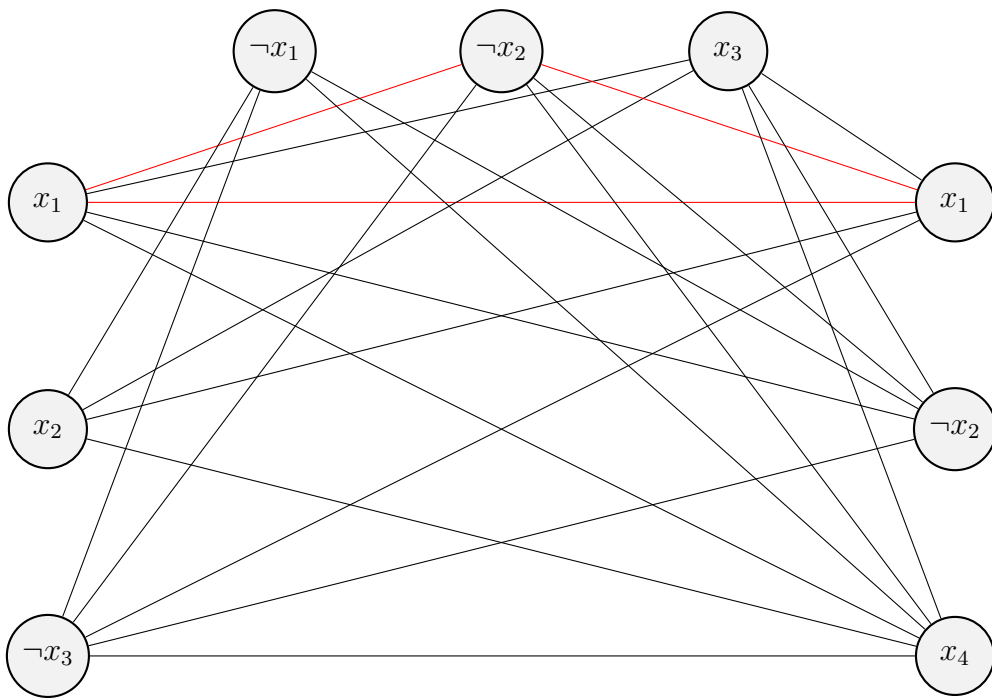
One 3-clique is shown by the three red edges containing the vertices x_1, $\neg x_2$, and x_1. So we need x_1 and $\neg x_2$ to be TRUE to satisfy the formula. Thus, we can set x_1 to TRUE and x_2 to FALSE.

2   Project Requirements

Write a program that will do the following:
Input: A Boolean formula in the form of a string.

An example would be "x_1 x_2 nx_3 nx_1 nx_2 x_3 x_1 nx_2 x_4" which would represent the formula in the previous section. nx_i represents $\neg x_i$.

Output: E, the number of edges in the graph formed using the reduction procedure outlined above.

1. Your program must accept a SINGLE ARGUMENT (the formula as a string) in the command line and it must print E and a new line (not return E) to standard output.

2. Your program must be written in Python 3 and called sat_to_cliq.py.

3. The variables range from x_1 to x_4.

4. There are 20 test cases. Each test case is a string of 9 terms, representing a Boolean formula with three clauses and three variables in each clause. The first three terms belong to the first clause, the middle three terms belong to the second, and the final three belong to the third.

5. You may use the sys and itertools libraries but no other Python libraries.

6. Submit your single sat_to_cliq.py file to Gradescope. You have unlimited submissions until the dead cat deadline but we will only run tests on your most recent submission.

7. As a sanity check, running
python sat_to_cliq.py "x_1 x_2 nx_3 nx_1 nx_2 x_3 x_1 nx_2 x_4"
should yield 22.

8. You may find the product function in itertools useful.

9. Here is a link to the itertools documentation:
https://docs.python.org/3/library/itertools.html#itertools.product

10. You must submit your own code.