BILL AND NATHAN START RECORDING

CFL $\subset$ P

Problem

Let L be a CFL. Let G be the Chomsky Normal Form CFG for L.

Problem

Let L be a CFL. Let G be the Chomsky Normal Form CFG for L.

$w = \sigma_1 \cdots \sigma_n$.

Problem

Let L be a CFL. Let G be the Chomsky Normal Form CFG for L.

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.

Problem

Let L be a CFL. Let G be the Chomsky Normal Form CFG for L.

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.
We want the algorithm to halt in $p(n)$ time for some polynomial.

Problem

Let L be a CFL. Let G be the Chomsky Normal Form CFG for L.

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.
We want the algorithm to halt in $p(n)$ time for some polynomial.
We will obtain $p(n) = O(n^3)$.

Omitting a Case

We will assume that $e \notin L$ and hence we do not have the rule

$$S \rightarrow e.$$

By the definition of Chomsky Normal Form, there are no other e-rules, hence there are no e-rules.

Omitting a Case

We will assume that $e \notin L$ and hence we do not have the rule

$$S \to e.$$

By the definition of Chomsky Normal Form, there are no other e-rules, hence there are no e-rules.

Our proof can be modified to accommodate this case.

Plan for CFL $\subset$ P

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.

Plan for CFL $\subset$ P

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.

We ask this differently:

$$\mathrm{GEN}(\sigma_1 \cdots \sigma_n) = \{A : A \Rightarrow \sigma_1 \cdots \sigma_n\}$$

Plan for CFL $\subset$ P

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.

We ask this differently:

$$\mathrm{GEN}(\sigma_1 \cdots \sigma_n) = \{A : A \Rightarrow \sigma_1 \cdots \sigma_n\}$$

We want to know if $S \in \mathrm{GEN}(\sigma_1 \cdots \sigma_n)$.

Plan for CFL $\subset$ P

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.

We ask this differently:

$$\mathrm{GEN}(\sigma_1 \cdots \sigma_n) = \{A : A \Rightarrow \sigma_1 \cdots \sigma_n\}$$

We want to know if $S \in \mathrm{GEN}(\sigma_1 \cdots \sigma_n)$.

I am tired of writing $\sigma_1 \cdots \sigma_n$.

Plan for CFL $\subset$ P

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.

We ask this differently:

$$\mathrm{GEN}(\sigma_1 \cdots \sigma_n) = \{A : A \Rightarrow \sigma_1 \cdots \sigma_n\}$$

We want to know if $S \in \mathrm{GEN}(\sigma_1 \cdots \sigma_n)$.

I am tired of writing $\sigma_1 \cdots \sigma_n$.

$$\mathrm{GEN}[1, n] = \{A : A \Rightarrow \sigma_1 \cdots \sigma_n\}$$

Plan for CFL $\subset$ P

$w = \sigma_1 \cdots \sigma_n$.
We want to know if $w \in L$.

We ask this differently:

$$\mathrm{GEN}(\sigma_1 \cdots \sigma_n) = \{A : A \Rightarrow \sigma_1 \cdots \sigma_n\}$$

We want to know if $S \in \mathrm{GEN}(\sigma_1 \cdots \sigma_n)$.

I am tired of writing $\sigma_1 \cdots \sigma_n$.

$$\mathrm{GEN}[1, n] = \{A : A \Rightarrow \sigma_1 \cdots \sigma_n\}$$

We are really asking: Is $S \in \mathrm{GEN}[1, n]$?

Easier to Solve a Harder Problem

All we want to know is:

is S in $\mathrm{GEN}[1, n]$?

Easier to Solve a Harder Problem

All we want to know is:

$$\text{is } S \text{ in } \mathrm{GEN}[1, n]?$$

I will solve a harder problem:

$$\text{Find } \mathrm{GEN}[1, n]$$

Easier to Solve an Even Harder Problem

For $i \leq j$ let

$$\mathrm{GEN}[i,j] = \{A : A \Rightarrow \sigma_i \cdots \sigma_j\}$$

Easier to Solve an Even Harder Problem

For $i \leq j$ let

$$\mathrm{GEN}[i,j] = \{A : A \Rightarrow \sigma_i \cdots \sigma_j\}$$

We will find **all** $\mathrm{GEN}[i,j]$.

Easier to Solve an Even Harder Problem

For $i \le j$ let

$$\mathrm{GEN}[i,j] = \{A : A \Rightarrow \sigma_i \cdots \sigma_j\}$$

We will find **all** $\mathrm{GEN}[i,j]$.

Hence we will find $\mathrm{GEN}[1,n]$.

Easier to Solve an Even Harder Problem

For $i \le j$ let

$$\mathrm{GEN}[i,j] = \{A : A \Rightarrow \sigma_i \cdots \sigma_j\}$$

We will find **all** $\mathrm{GEN}[i,j]$.

Hence we will find $\mathrm{GEN}[1,n]$.

Hence we will find if $S \in \mathrm{GEN}[1,n]$.

Easier to Solve an Even Harder Problem

For $i \le j$ let

$$\mathrm{GEN}[i,j] = \{A : A \Rightarrow \sigma_i \cdots \sigma_j\}$$

We will find **all** $\mathrm{GEN}[i,j]$.

Hence we will find $\mathrm{GEN}[1,n]$.

Hence we will find if $S \in \mathrm{GEN}[1,n]$.

Why solve this harder problem?

Easier to Solve an Even Harder Problem

For $i \le j$ let

$$\mathrm{GEN}[i,j] = \{A : A \Rightarrow \sigma_i \cdots \sigma_j\}$$

We will find **all** $\mathrm{GEN}[i,j]$.

Hence we will find $\mathrm{GEN}[1,n]$.

Hence we will find if $S \in \mathrm{GEN}[1,n]$.

Why solve this harder problem?

We will use Dynamic programming so having some $\mathrm{GEN}[i,j]$ solved will help us solve later ones.

$\mathrm{GEN}[i,i]$ and $\mathrm{GEN}[i,i+1]$

We find $\mathrm{GEN}[i,i]$.

$\mathrm{GEN}[i, i]$ and $\mathrm{GEN}[i, i+1]$

We find $\mathrm{GEN}[i, i]$.

$$\mathrm{GEN}[i, i] = \{A : A \to \sigma_i\}$$

$\mathrm{GEN}[i,i]$ and $\mathrm{GEN}[i,i+1]$

We find $\mathrm{GEN}[i,i]$.

$$\mathrm{GEN}[i,i] = \{A : A \rightarrow \sigma_i\}$$

We find $\mathrm{GEN}[i,i+1]$.
The only way for $A \Rightarrow \sigma_i\sigma_{i+1}$ is if
$A \rightarrow BC$
$B \rightarrow \sigma_i$ (AH- then $B \in \mathrm{GEN}[i,i]$.)
$C \rightarrow \sigma_{i+1}$ (AH- then $C \in \mathrm{GEN}[i+1,i+1]$).

$\mathrm{GEN}[i,i]$ and $\mathrm{GEN}[i,i+1]$

We find $\mathrm{GEN}[i,i]$.

$$\mathrm{GEN}[i,i] = \{A : A \rightarrow \sigma_i\}$$

We find $\mathrm{GEN}[i,i+1]$.
The only way for $A \Rightarrow \sigma_i\sigma_{i+1}$ is if
$A \rightarrow BC$
$B \rightarrow \sigma_i$ (AH- then $B \in \mathrm{GEN}[i,i]$.)
$C \rightarrow \sigma_{i+1}$ (AH- then $C \in \mathrm{GEN}[i+1,i+1]$).
$\mathrm{GEN}[i,i+1] =$

$$\{A : A \rightarrow BC \quad \wedge \quad B \in \mathrm{GEN}[i,i] \quad \wedge \quad C \in \mathrm{GEN}[i+1,i+1]\}$$

$\mathrm{GEN}[i, i+2]$

We find $\mathrm{GEN}[i, i+2]$.
The only way for $A \Rightarrow \sigma_i \sigma_{i+1} \sigma_{i+2}$ is if
EITHER:
$A \rightarrow BC$
$B \rightarrow \sigma_i$ (AH- then $B \in \mathrm{GEN}[i, i]$.)
$C \Rightarrow \sigma_{i+1} \sigma_{i+2}$ (AH- then $C \in \mathrm{GEN}[i+1, i+2]$).

$\mathrm{GEN}[i, i+2]$

We find $\mathrm{GEN}[i, i+2]$.
The only way for $A \Rightarrow \sigma_i \sigma_{i+1} \sigma_{i+2}$ is if
EITHER:
$A \rightarrow BC$
$B \rightarrow \sigma_i$ (AH- then $B \in \mathrm{GEN}[i, i]$.)
$C \Rightarrow \sigma_{i+1} \sigma_{i+2}$ (AH- then $C \in \mathrm{GEN}[i+1, i+2]$).
OR:

$\mathrm{GEN}[i, i+2]$

We find $\mathrm{GEN}[i, i+2]$.
The only way for $A \Rightarrow \sigma_i\sigma_{i+1}\sigma_{i+2}$ is if
EITHER:
$A \rightarrow BC$
$B \rightarrow \sigma_i$ (AH- then $B \in \mathrm{GEN}[i, i]$.)
$C \Rightarrow \sigma_{i+1}\sigma_{i+2}$ (AH- then $C \in \mathrm{GEN}[i+1, i+2]$).

OR:

$A \rightarrow BC$
$B \Rightarrow \sigma_i\sigma_{i+1}$ (AH- then $B \in \mathrm{GEN}[i, i+1]$.)
$C \rightarrow \sigma_{i+2}$ (AH- then $C \in \mathrm{GEN}[i+2, i+2]$).

$\mathrm{GEN}[i, i+2]$

We find $\mathrm{GEN}[i, i+2]$.
The only way for $A \Rightarrow \sigma_i \sigma_{i+1} \sigma_{i+2}$ is if
EITHER:
$A \rightarrow BC$
$B \rightarrow \sigma_i$ (AH- then $B \in \mathrm{GEN}[i, i]$.)
$C \Rightarrow \sigma_{i+1}\sigma_{i+2}$ (AH- then $C \in \mathrm{GEN}[i+1, i+2]$).

OR:

$A \rightarrow BC$
$B \Rightarrow \sigma_i \sigma_{i+1}$ (AH- then $B \in \mathrm{GEN}[i, i+1]$.)
$C \rightarrow \sigma_{i+2}$ (AH- then $C \in \mathrm{GEN}[i+2, i+2]$).
We can write $\mathrm{GEN}[i, i+2]$ in terms of $\mathrm{GEN}[i, i]$, $\mathrm{GEN}[i, i+1]$, $\mathrm{GEN}[i+1, i+2]$.

$\mathrm{GEN}[i, i+2]$

We find $\mathrm{GEN}[i, i+2]$.
The only way for $A \Rightarrow \sigma_i\sigma_{i+1}\sigma_{i+2}$ is if
EITHER:
$A \rightarrow BC$
$B \rightarrow \sigma_i$ (AH- then $B \in \mathrm{GEN}[i, i]$.)
$C \Rightarrow \sigma_{i+1}\sigma_{i+2}$ (AH- then $C \in \mathrm{GEN}[i+1, i+2]$).

OR:

$A \rightarrow BC$
$B \Rightarrow \sigma_i\sigma_{i+1}$ (AH- then $B \in \mathrm{GEN}[i, i+1]$.)
$C \rightarrow \sigma_{i+2}$ (AH- then $C \in \mathrm{GEN}[i+2, i+2]$).
We can write $\mathrm{GEN}[i, i+2]$ in terms of $\mathrm{GEN}[i, i]$, $\mathrm{GEN}[i, i+1]$, $\mathrm{GEN}[i+1, i+2]$.

More important: we can easily FIND $\mathrm{GEN}[i, i+2]$ if we KNOW $\mathrm{GEN}[i, i]$, $\mathrm{GEN}[i, i+1]$, $\mathrm{GEN}[i+1, i+2]$.

$\mathrm{GEN}[i, i+k]$

We find $\mathrm{GEN}[i, i+k]$.
The only way for $A \Rightarrow \sigma_i \cdots \sigma_{i+k}$ is if there exists j such that
$A \rightarrow BC$
$B \Rightarrow \sigma_i \cdots \sigma_j$ (AH- then $B \in \mathrm{GEN}[i, j]$.)
$C \Rightarrow \sigma_{j+1} \sigma_k$ (AH- then $C \in \mathrm{GEN}[j+1, k]$).

$\mathrm{GEN}[i, i+k]$

We find $\mathrm{GEN}[i, i+k]$.
The only way for $A \Rightarrow \sigma_i \cdots \sigma_{i+k}$ is if there exists j such that
$A \rightarrow BC$
$B \Rightarrow \sigma_i \cdots \sigma_j$ (AH- then $B \in \mathrm{GEN}[i, j]$.)
$C \Rightarrow \sigma_{j+1} \sigma_k$ (AH- then $C \in \mathrm{GEN}[j+1, k]$).

Can use this recurrence bottom up to get a DYN PROGRAM for the problem

The Algorithm

1) For $i = 1$ to n $\mathrm{GEN}[i, i] = \{A : A \to \sigma_i\}$

The Algorithm

1) For $i = 1$ to n $\mathrm{GEN}[i, i] = \{A : A \to \sigma_i\}$
2) For $k = 1$ to $n - 1$ we will look at $\mathrm{GEN}[i, i + k]$
   For $i = 1$ to $n - k$
   $\mathrm{GEN}[i, i + k] =$

$$\bigcup_{i<j<k} \{A : A \to BC \quad \wedge \quad B \in \mathrm{GEN}[i, j] \quad \wedge \quad C \in \mathrm{GEN}[j + 1, k]\}$$

3) If $S \in \mathrm{GEN}[1, n]$ then output YES, else output NO.