BILL AND NATHAN START RECORDING

Context Sensitive Languages

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5) Languages that are CSL but not CFL.

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1. While human language is far more complicated than CFL or CSL; the Mathematical tools these grammars supply were a helpful starting point.
2. Computer languages are far easier to understand since we make them ourselves; hence, CFLs and (to a lesser extend) CSL's were useful within Computer Science.

Examples of Context Sensitive Grammars

$S \to ABCS \quad | \quad e$

$AB \to BA$ (Note- We allow two nonterminals on the LHS.)

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3) Context-Sensitive means can replace (say) A by (say) α AND look at what is around A. We actually allow more than that.

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$S \to abc \mid aXbc \mid e$
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Note It's a real mess to prove, and not that intuitive.

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Upshot CSG's are messy and we are not going to deal with them much.

Definition of Context Sensitive Grammars

Def A **Context Sensitive Grammar** is a tuple $G = (N, \Sigma, R, S)$

- N is a finite set of **nonterminals**.
- Σ is a finite **alphabet**. Note $\Sigma \cap N = \emptyset$.
- $R \subseteq (N \cup \Sigma)^* N (N \cup \Sigma)^* \times (N \cup \Sigma)^*$ and are called **Rules**.
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Note

1) The LHS must have at least one nonterminal.

2) There are alternative definitions that are equivalent, which I won't get into here.

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Then, if w is string of **non-terminals only**, we define $L(G)$ by:

$$L(G) = \{w \in \Sigma^* \mid S \Rightarrow w\}$$

Example of a Lang that is NOT a CSL

We'll come back to this later.

CLOSURE PROPERTIES

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The proof that LBA-recognizers and CSG-generators are equivalent is messy so we won't be doing it. We won't deal with LBA's in this course at all.

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Hence it is easy to show that $\{a^{n^2} : n \in \mathrm{N}\}$ and many other languages are CSL's without using CSG's.

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Open question Some variants of Chess and Go **might be** provably not CSL.

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