BILL, RECORD LECTURE!!!!

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A Lang that has a TINY CFG a MEDIUM NFA and a LARGE DFA

DFA and NFA and CFG

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2. Every Chomsky Normal Form CFG for L_n has $\geq \lg n$ size. We will not prove this.

Upper and Lower Bounds for DFA for L_n

Upper and Lower Bounds for NFA for L_n

Upper and Lower Bounds for CFG for L_n

CFG for Σ^*

Lemma There is a CFG for Σ^* of size $O(1)$.

$S \rightarrow AS$

$S \rightarrow BS$

$S \rightarrow e$

$A \rightarrow a$

$B \rightarrow b$

CFG for Σ^n of Size $\le 2\lg n$

We first do this when $n = 2^m$, a power of 2.

$S \to A_1 A_1$

$A_1 \to A_2 A_2$

$A_2 \to A_3 A_3$

$\vdots$

$A_{m-1} \to A_m A_m$

$A_m \to a \mid b$

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Can show that
$S \Rightarrow A_1A_1 = A_1^2 = A_1^{2^1}$
$S \Rightarrow A_2A_2A_2A_2 = A_2^4 = A_2^{2^2}$
$(\forall 1 \leq i \leq m)[S \Rightarrow A_i^{2^i}]$
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Since A_m is the only nonterminal that can generate terminals, and A_m^n is the only string of A_m's that can be produced, and $A_m \to a \mid b$, we have

CFG for Σ^n

What if n is not a power of 2?
Leave it to you to show there is a Chomsky Normal Form Grammar of Size $\leq 2 \lg n$.

CFG for $\Sigma^* a \Sigma^n$

Let G_1 be grammar for Σ^*. Let S_1 be its start symbol. Size 5
Let G_2 be grammar for Σ^n. Let S_2 be its start symbol. Size $\le 2 \lg n$.
Relabel if needed so they use disjoint set of nonterminals.

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Size $\leq 3 + 5 + 2 \lg n = 8 + 2 \lg n$.