BILL AND NATHAN RECORD LECTURE!!!!

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BILL AND NATHAN RECORD LECTURE!!!

UN-TIMED PART OF FINAL IS TUESDAY May 11 11:00A. NO DEAD CAT

FINAL IS THURSDAY May 13 8:00PM-10:15PM

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FILL OUT COURSE EVALS for ALL YOUR COURSES!!!

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Other Topics I Could Have Covered And Might Next Spring

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Other Topics I Could Have Covered And Might Next Spring

Exposition by William Gasarch—U of MD

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Steps Forward and Backwards

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A Step Backwards means an old topic, we'll say pre-1980. Often Logic or more tied to the actual machine model. This is not necc bad.

Topics on Reg Langs

Exposition by William Gasarch—U of MD

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1. Pattern Matching



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- 2. Perl-Regular, Ruby-Regular, etc.

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Verdict Have not done. Perl-Regular might drive me nuts since it does not have a clean mathematical semantics.

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Desc of Reg Expressions

Theorems about lower bounds on lengths of Regular Expressions.

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Theorems about lower bounds on lengths of Regular Expressions. **Verdict** Would have to learn those theorems, which I want to. https:

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Topics on CFL's

Exposition by William Gasarch—U of MD

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1. PDA's are DFA's with a stack and are use to model compilers.

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2. Applications of CFG's and PDA's to Compiler design

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- 3. C++ syntax is undecidable

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- 3. C++ syntax is undecidable

Verdict Won't be covering. Too messy. Will mention these aspects more than I did.

Kudos to the person who told me that C++ syntax is undecidable. Good to know!

All papers on this are here: https:

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1. Deterministic PDA's which play into length of descriptions.

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- 1. Deterministic PDA's which play into length of descriptions.
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Topics on Complexity Theory

Exposition by William Gasarch—U of MD

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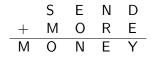
Verdict I should write a parody of Aretha Franklin's song RESPECT with this theme.

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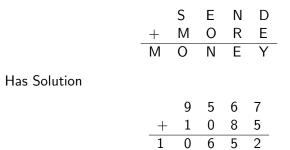
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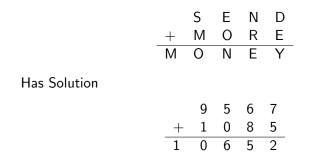
Also, would be happy to do any of these topics.



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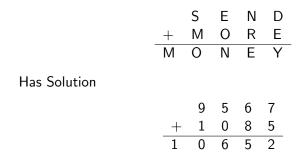


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Given a puzzle, does it have a solution, is NP-complete

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Given a puzzle, does it have a solution, is NP-complete Verdict Not sure. Good to see one hard reduction. Too hard?

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Def A *c*-coloring of an $n \times m$ grid is a coloring that has no monochromatic rectangles.

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I would know- it was my open problem and I am an author on the paper that solved it.

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1. CHESS is EXPTIME-complete



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- 3. Equiv of trex is EXPSPACE-complete

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- 1. CHESS is EXPTIME-complete
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Verdict trex ties into the other parts of the course. But all of these proof are similar to Cook-Levin so messy TM stuff. A Step Backwards.

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 \exists sets decidable in poly time with 5 queries to SAT but not 4? Assuming $\Sigma_2^p\neq\Pi_2^p,$ YES.

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∃ sets decidable in poly time with 5 queries to SAT but not 4? Assuming $\Sigma_2^p \neq \Pi_2^p$, YES. Verdict Number of queries as a complexity measure is interesting. Would be happy to do these topics.

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- 2. Would take 2 or 3 lectures.

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1. A set A is sparse if \exists poly p, $|A \cap \Sigma^n| \le p(n)$.

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Verdict I have done both of these in class and may do it again. A tiny step backwards.

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1. Nondet-Log-Space is closed under complement.

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- 2. Nondet-Log-Space is contained in P.
- **3**. NSPACE(S(n)) \subseteq DSPACE($S(n)^2$).

Verdict All nice theorems that I could do. Would need to introduce and talk about space complexity so this would take time. Not that hard, so thats good.

Caveat Space Complexity is not as much fun as a theme as RESPECT is.

Primitive Recursive Functions

Def Prim Rec Functions are in between P and Undecidable.

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Def Prim Rec Functions are in between P and Undecidable. **What They Include** Exp, double-exp, Tower, WOWER, etc. **Where Used** In some branches of Math Prim-rec vs non-prim-rec is like P vs EXP for CS. Notably in Ramsey Theory. **Verdict** Number of Queries as a complexity measure is interesting and could be a theme for the course.

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Decidable and Undecidable

Exposition by William Gasarch—U of MD

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1. Presburger Arithmetic is decidable: just < and +.

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- 3. S2S is way to hard!

1. Presburger Arithmetic is decidable: just < and +.

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- 4. Theory of the reals is decidable!

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Verdict A step Backwards.

 \exists fnctns computable with 5 queries to HALT but not 4?

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 \exists fnctns computable with 5 queries to HALT but not 4? \exists sets computable with 5 queries to HALT but not with 4?

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 \exists fnctns computable with 5 queries to HALT but not 4? \exists sets computable with 5 queries to HALT but not with 4? Yes. No assumption needed.

∃ fnctns computable with 5 queries to HALT but not 4?
∃ sets computable with 5 queries to HALT but not with 4?
Yes. No assumption needed.
Verdict Draws on my own research, so I care. Do you?

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1. Given a CFG G, is $\overline{L(G)}$ a CFL? Undecidable. Could actually prove this.

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- PowerPoint is Undecidable: There is a reduction from HALT to POWERPOINT meaning that if x ∈ HALT then there will be one slides with a YES, and if x ∉ HALT then there will be one slide with a NO. Interesting but too complicated.

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4. Actually prove Hilbert's tenth is undecidable. Too complicated.

Verdict The first one is plausible, but a step backwards.

Let T be a theory (e.g., Presburger plus \times).

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Analog

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WOW There are statements that are true but not provable!

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Analog

WOW There are statements that are true but not provable! is like saying

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WOW There are statements that are true but not provable! is like saying WOW Women can vote!

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Analog

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is like saying

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Both are true but neither is surprising anymore.

Verdict Really not sure about this one. Would need to give context and history, but a very important theorem.

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Tori and Guido in 2036

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Tori and Guido in 2036

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Lets make the statements

Tori and Guido in 2036

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Lets make the statements

WOW Women can be president!

Tori and Guido in 2036

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Lets make the statements

WOW Women can be president!

WOW Non-citizen's can be vice-president!

Tori and Guido in 2036

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Lets make the statements

WOW Women can be president!

WOW Non-citizen's can be vice-president!

True and not surprising.

Arithmetic Hierarchy

Actually **prove** that (say)

 $\mathrm{INF} = \{e: \textit{M}_e \text{ halts on an infinite number of numbers}\}$ is NOT in $\Sigma_2.$

Arithmetic Hierarchy

Actually **prove** that (say)

$$\label{eq:INF} \begin{split} \mathrm{INF} &= \{e: \mathit{M}_e \text{ halts on an infinite number of numbers} \} \\ \text{is NOT in } \Sigma_2. \\ \textbf{Verdict Too much background and a step backwards.} \end{split}$$

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Are there sets that are both



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1. Not decidable

Are there sets that are both

- 1. Not decidable
- 2. Weaker than HALT.

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Are there sets that are both

- 1. Not decidable
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Answer: YES and the proof is interesting but hard.

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Are there sets that are both

- 1. Not decidable
- 2. Weaker than HALT.

Answer: YES and the proof is interesting but hard. **Verdict** A step backwards but a very interesting proof.

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I could apply Kolm Complexity to



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1. Proving more langs not regular.

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I could apply Kolm Complexity to

- 1. Proving more langs not regular.
- 2. Proving some langs have large DFAs, NFAs, CFGs.

I could apply Kolm Complexity to

- 1. Proving more langs not regular.
- 2. Proving some langs have large DFAs, NFAs, CFGs.

3. Getting Avg Case Analysis of some algorithms.



Exposition by William Gasarch—U of MD



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1. Muffin problems have upper and lower bounds that **match**. A good example of what we WANT to be able to achieve in complexity.

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1. Muffin problems have upper and lower bounds that match. A good example of what we WANT to be able to achieve in complexity.

2. My Muffin-Math song:

https://www.youtube.com/watch?v=4xQFlsK7jKg is the 2nd worse math song in Youtube. The worst is

- Muffin problems have upper and lower bounds that match. A good example of what we WANT to be able to achieve in complexity.
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Verdict I want to teach Muffin-Math, Muffin-Math, Muffin-Math, I want to teach Muffin-Math, the answer is 5/12.

Scenario Alice has $x \in \{0, 1\}^n$. Bob has $y \in \{0, 1\}^n$.

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Scenario Alice has $x \in \{0, 1\}^n$. Bob has $y \in \{0, 1\}^n$. They want to know if x = y.

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Scenario Alice has x ∈ {0,1}ⁿ. Bob has y ∈ {0,1}ⁿ. They want to know if x = y.
Alice could just say Hey Bob, my string is x
That would take n bits of communication.
Can they do better? Vote.
1) YES they can and this is known.
2) NO they can't and this is known.
3) UNKNOWN TO SCIENCE.
NO they can't and this is known.

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Communication Complexity

Scenario Alice has x ∈ {0,1}ⁿ. Bob has y ∈ {0,1}ⁿ. They want to know if x = y.
Alice could just say Hey Bob, my string is x
That would take n bits of communication.
Can they do better? Vote.
1) YES they can and this is known.
2) NO they can't and this is known.
3) UNKNOWN TO SCIENCE.
NO they can't and this is known.

Can use results in Comm Complexity to show langs are not regular.

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Communication Complexity

Scenario Alice has $x \in \{0, 1\}^n$. Bob has $y \in \{0, 1\}^n$. They want to know if x = y. Alice could just say **Hey Bob**, my string is x That would take *n* bits of communication. Can they do better? Vote. 1) YES they can and this is known. NO they can't and this is known. 3) UNKNOWN TO SCIENCE. NO they can't and this is known. Can use results in Comm Complexity to show langs are not regular. Verdict Have done, could do again. A step forward.

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There are other modes of computation.

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1. Parallelism: There is a theory analogous to P vs NP to show problems can't be parallelized.

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- 2. Randomized Computations: How much does randomization help?
- 3. Quantum Computing: there is a notion of quantum-DFA that I could look into and do, but might be too hard. For me!

Verdict I would have to look into all of these more to see if they make sense. Quantum would be a step forward.

Imagine if we did not have Cook-Levin but still thought SAT was hard.

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Imagine if we did not have Cook-Levin but still thought SAT was hard.

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There are other groups of problems where this IS what we have.

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- 2. $CLIQ_k$ seems to REQUIRE $n^{\Omega(k)}$ time. There are now $CLIQ_k$ -hard problems.
- 3. There are others.

What to take Out (Brief)

Exposition by William Gasarch—U of MD

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If I want to put any of that in, I need to take some stuff out.

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1. CFG's I could easily take out. :-)

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- Recursion Theory. NEED to prove HALT is undecidable. LIKE to prove WS1S is decidable. All else can go. Maybe even WS1S can go :-(

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- Recursion Theory. NEED to prove HALT is undecidable. LIKE to prove WS1S is decidable. All else can go. Maybe even WS1S can go :-(
- 3. Could reduce how much time I spend on regular by cutting out Regular Expressions. They are done in 330 anyway. DO want to keep the SMALL-NFA-RESPECT problem.

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- 4. Could do less HW review- only go over the problems student had trouble with.

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- 3. Could reduce how much time I spend on regular by cutting out Regular Expressions. They are done in 330 anyway. DO want to keep the SMALL-NFA-RESPECT problem.
- 4. Could do less HW review- only go over the problems student had trouble with.

5. Could go faster by making it a truly flipped classroom.

BILL AND NATHAN RECORD LECTURE!!!!

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BILL AND NATHAN RECORD LECTURE!!!

UN-TIMED PART OF FINAL IS TUESDAY May 11 11:00A. NO DEAD CAT

Exposition by William Gasarch—U of MD

FINAL IS THURSDAY May 13 8:00PM-10:15PM

Exposition by William Gasarch—U of MD

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