Homework 1 Morally Due Feb 6

1. (0 points) If you are not getting emails that the class gets, then email Bill as soon as possible.

2. (30 points) List all of the elements of $\{0, 1, \ldots, 20\}$ that have multiplicative inverses mod 21. For each such element also give the inverse.

3. (40 points) The alphabet is $\{0,\ldots,9\}$. We interpret the input as a base 10 natural number, read *right to left*. So the number 29139 will be read 9-3-1-9-2.

   (a) (10 points) Compute
   $10^0 \pmod{14}$
   $10^1 \pmod{14}$
   $10^2 \pmod{14}$
   etc.
   until you spot a pattern. What is the pattern?

   (b) (10 points) Recall that

   $$a_k \times 10^k + a_{k-1} \times 10^{k-1} + \cdots + a_0 \times 10^0 \equiv a_k + \cdots + a_0 \pmod{9}.$$

   Come up with a statement of that type for mod 14. You may use DOT-DOT-DOT and you may have a set of cases.

   (c) (20 points) Let

   $$A = \{x \colon x \equiv 5 \pmod{14}\} \cup \{x \colon x \equiv 7 \pmod{14}\}.$$

   (x is in base 10.)
   Give the DFA for A by giving the transition table and specifying what are the start state is and what the final states are.
   ADVICE For the transtion table **do not** have LOTS of rows. You can have things like (this is not the actual answer)
   For all $\sigma \in \{0,\ldots,9)$, $\delta(q,\sigma) = x + 8\sigma \pmod{87}$.
   There may be cases.

4. (30 points) We define $\#_a(w)$ to be the number of a's in w. The alphabet is $\{a, b\}$.

   (a) (15 points) Draw a DFA for $\{w \mid \#_a(w) \equiv 0, 1 \pmod 4\}$. How many states does this DFA have?

   (b) (15 points) Draw a DFA for $\{w \mid \#_a(w) \equiv 0, 2 \pmod 4\}$. How many states does this DFA have? (Hint: this should be less states than the prior part.)