Homework 9 Morally Due April 23 at 3:30PM

1. (30 points) In this problem sets are sets of natural numbers.

   Recall that

   $A \in \Sigma_1$ if there exists B decidable such that

   $$A = \{x : (\exists y)[B(x,y)]\}.$$

   Definition A is an ADAM SET if there exists a Turing Machine M with the following behaviour:

   - If $x \in A$ then $M(x)$ halts.
   - If $x \notin A$ then $M(x)$ does not halt.

   And NOW for the problem:

   (a) Show that if $A \in \Sigma_1$ then A is an ADAM set.

   (b) Show that if A is an ADAM set then $A \in \Sigma_1$.

2. (30 points) **Definition** Let $w \in \Sigma^*$. Then ISAAC(w) is the set of words that can be formed by removing any set of symbols from w. For example

$$\text{ISAAC}(abab) = \{e, a, b, aa, ab, ba, bb, aab, aba, abb, bab, abab\}$$

If L is a language (a subset of Σ^*) then

$$\text{ISAAC}(L) = \bigcup_{w \in L} \text{ISAAC}(w).$$

For example if $A = \{abab, bbbb\}$ then

$$\text{ISAAC}(A) = \{e, a, b, aa, ab, ba, bb, aab, aba, abb, bab, bbb, abab, bbbb\}$$

(a) (25 points) Show that if $L \in \Sigma_1$ then ISAAC(L) $\in \Sigma_1$. (You may use the quantifier definition of Σ_1 or the ADAM definition of Σ_1. Either one will work.)

(b) (5 points)

VOTE one of the following (Note: You do not need to vote correctly to receive points):

- If L is decidable then ISAAC(L) is decidable. Fire and Brimstone Speech to Follow.
- There exists an L that is decidable such that ISAAC(L) is NOT decidable.
- The question is UNKNOWN TO SCIENCE.

3. (40- 8 points each) You are designing an algorithm for CNFSAT. I will incompletely describe some short cuts you can take. Fill in the BLANK

   The input is of the form

   $C_1 \wedge \cdots \wedge C_m$

   where each C_i is an OR of literals (a literal is a var or its negation).

   (a) If $C_1 = (x_3)$ then you can do $BLANK_1$.

   (b) If x_4 appears in the formula but $\neg x_4$ never appears then you can do $BLANK_2$.

   (c) If $C_2 = (x_8)$ and $C_3 = (x_9)$ and $C_4 = (\neg x_8 \vee \neg x_9)$ then you can do $BLANK_3$.

   (d) If $C_4 = (x_{10} \vee \neg x_{11} \vee x_{12} \vee \neg x_{12})$ then you can do $BLANK_4$.

   (e) If there are no negation signs in the formula then you can do $BLANK_5$