Homework 10 Morally Due April 30 at 3:30PM

1. (30 points)

   (a) (30 points) Let f be a computable function from N to N such that

   $$(\forall x, y)[x < y \rightarrow f(x) < f(y)].$$

   (so f is increasing).
   Show that the set of numbers in the image of f is decidable. Formally the image is

   $$\{y : (\exists x)[f(x) = y]\}.$$

   (b) (0 points but I want you to think about this one)
   Let f be a computable function from N to N such that

   $$(\forall x, y)[x < y \rightarrow f(x) \leq f(y)].$$

   (so f is monotonically increasing).
   IS the image of f decidable? THINK ABOUT IT.

2. (30 points) Show that the following problem is DECIDABLE:

   Given a polynomial $p(x) = a_n x^n + \cdots + a_0$ where $a_n, \ldots, a_0 \in \mathsf{Z}$. determine if p have a root in Z.

   (**Hint** Your first step is to rewrite $p(x)$ as

   $p(x) = x(a_n x^{n-1} + \cdots + a_1) + a_0$.

   Your next step is to set this to 0 and see if that bounds what x can be.

3. (40 points)

   (a) (10 points) Using the WS1S convention give a DFA for

   $$\{(x, y) : x = y + 1\}.$$

   How many states does your DFA have?
   (All states are labelled A for accept or R for reject or S for stupid.)

   (b) (10 points) Using the WS1S convention give a DFA for

   $$\{(x, y) : x = y + 2\}.$$

   How many states does your DFA have?
   (All states are labelled A for accept or R for reject or S for stupid.)

   (c) (20 points) Let $a \in \mathsf{N}$ and $a \geq 1$. Using the WS1S convention give a DFA for

   $$\{(x, y) : x = y + a\}.$$

   You will need to use DOT DOT DOT.
   How many states does your DFA have have as a function of a?
   (All states are labelled A for accept or R for reject or S for stupid.)