Optional Project Morally Due May 7 3:30PM

This is an OPTIONAL PROJECT. It would be more accurate to say its an OPTIONAL HW. I will not look at it until after the final is graded and the final grades have been determined.

1. If you have a D in the course and do a VERY GOOD JOB on the optional project then I will bump your grade to a C-.

2. If you have an F in the course (note- in my 40 years of teaching this has only happened twice) and you do a VERY GOOD JOB on the optional project then I will bump your grade to a D.

3. The following has actually happened. DON"T BE THIS GUY. A guy does badly on the midterm, does not do the project, gets a D in the course, and THEN asks me if he could do some kind of optional project to bring his grade up to a C. The answer was of course NO. DO NOT BE THAT GUY.

4. Students often ask me for sample problems. Consider this to be a set of sample problems whether you do it or not.

5. Should you do this if you are in no danger of failing the course? Yes- its a good study aid.

6. Can this bump your grade from a C to a B or a B to an A or some such? NO. I will only look at those from the D and F students.

7. Start the project NOW. You don't want to do it at the last minute.

1. (15 points)

   Let L_1 be regular with DFA $(Q_1, \Sigma, \delta_1, s_1, F_1)$.

   Let L_2 be regular with DFA $(Q_2, \Sigma, \delta_2, s_2, F_2)$.

   Let \$ be a symbol that is not in Σ.

   Construct a DFA for the language

$$\{x\$y : x \notin L_1 \wedge y \notin L_2\}.$$

2. (15 points) Let G be the grammar with rules

   $S \to AB$

   $A \to BA$

   $B \to SA$

   $A \to a$

   $B \to b$.

   Let $L = L(G)$.

   In this problem you will, by hand, do the algorithm for $L \in P$ on the string *abb*.

   Do by hand the algorithm for $L \in P$ on the input *abba*. In particular, fill in the BLANKS below, explain your work, and determine if $abba \in L$.

   GEN$[1,1]$ = BLANK.

   GEN$[2,2]$ = BLANK.

   GEN$[3,3]$ = BLANK.

   GEN$[4,4]$ = BLANK.

   GEN$[1,2]$ = BLANK.

   GEN$[2,3]$ = BLANK.

   GEN$[3,4]$ = BLANK.

   GEN$[1,3]$ = BLANK.

   GEN$[2,4]$ = BLANK.

   GEN$[1,4]$ = BLANK.

   Since GEN$[1,4]$ BLANK, we know that the question $abba \in L$? has the answer BLANK.

3. (15 points) Show that if L is in P then L^* is in P.

4. (20 points- 1 point each) Assume P $\neq$ NP. The following is a list of problems that are in NP. For each one state which of the following holds:

   - The problem is known to be in P
   - The problem is known to NOT be in P.
   - It is NOT KNOWN if the problem is in P.

   NOW, here is the list:

   (a) $\{(G,k) : G$ is k-colorable $\}$.

   (b) $\{G : G$ is 1000-colorable $\}$.

   (c) $\{(G,k) : G$ has a Vertex Cover of size k $\}$.
   (Recall that a *Vertex Cover of* $G = (V,E)$ is a set $U \subseteq V$ such that every EDGE has at least one vertex in U as an endpoint.

   (d) $\{G : G$ has a Vertex Cover of size ≤ 1000 $\}$.

   (e) $\{(G,k) : G$ has a Dom Set of size k $\}$.
   (Recall that a *Dom Set of* $G = (V,E)$ is a set $U \subseteq V$ such that every VERTEX has at least one vertex in U as a NEIGHBOR.)

   (f) $\{G : G$ has a Dom Set of size ≤ 1000 $\}$.

   (g) $\{n : (\exists n_1, n_2, n_3, n_4 \in \mathsf{N})[\mathsf{n} = \mathsf{n}_1^2 + \mathsf{n}_2^2 + \mathsf{n}_3^2 + \mathsf{n}_4^2]\}$
   (Note that n is in binary. Note that N includes 0. These hold for the next two problem as well.)

   (h) $\{n \in \mathsf{N} : \mathrm{n}$ is prime$\}$

   (i) $\{(n,m) : n$ has a factor that is $\leq m\}$

   (j) $\{(G,H) : G$ and H are isomorphic$\}$

5. (20 points) This problem uses WS1S notation. Draw the DFA for

$$\{(x, X) : x + 2 \in X\}.$$

(NOTE: I posted this optional project on April 4. I had not covered the material needed to do this problem yet.)

6. (15 points) Present 5 sets that are not decidable. (PRESENT THEM CLEARLY!)

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