CMSC 452 Project
Morally due March 12 by 3:30pm

1 Small NFAs for L_n

Let L_n be defined as

$$L_n = \{a^i : i \neq n\}.$$

From the slides you know that there are NFAs for L_n of size much smaller than n. How do you find such an NFA? How do you represent it? Here is the procedure:

1. Find (x, y) relatively prime such that $m = xy - x - y \leq n$, but you want to make m not too much smaller than n. We will formalize this as $0 \leq n - (xy - x - y) \leq 4\sqrt{n}$. We will also take $x < y$.

2. M is an NFA with (1) a chain of size $c = n - m$ from the start state to a final state q, (2) a loop around q of size y, and (3) a transition from the xth state in the loop back to state q.

   Note that q is considered part of the big loop and not part of the chain, but that the start state is considered part of the chain.

3. ε-transitions to states that have loops of size $p_1, \ldots, p_\ell$ (where the p_i's are all prime) where

   $$p_1 \times \cdots \times p_\ell \geq n$$

   but you want to keep $p_1 + \cdots + p_\ell$ small. One easy way is to just take the least ℓ such that

   $$p_1 \times \cdots \times p_\ell \geq n$$

   and

   $$p_1 \times \cdots \times p_{\ell-1} < n.$$

4. On those loops you need to have as final states all states corresponding to $\not\equiv n \pmod{p_i}$.

So the only parameters you need to specify the NFA are

- x, y
- c, the size of the chain
- $p_1, \ldots, p_\ell$ primes

The number of states s in the NFA is $s = \max(x, y) + c + p_1 + \cdots + p_\ell$.

2 Project Requirements

Write a program that will do the following:
Input: Integer n
Output: The parameters $(x, y, c, \ell, p_1, \ldots, p_\ell)$ and the number of states s in your NFA. (The number of states must be strictly less than n.
Constraint: $100 \leq n \leq 10^{12}$

1. Your program may be written in any programming language supported by Codeforces.
2. You are permitted to use any built-in libraries of your chosen language.
3. Create an account on **Codeforces**, click on this **link**, and submit your solution to the specified problem.
4. The assessment consists of 200 tests; for each test passed, you will receive a score of 0.5.
5. Submissions must be your original work.
6. Submissions will be automatically graded, and results will be available immediately after submission.
7. Upload a screenshot of your submission, showing the grade you are satisfied with, to Gradescope (submissions will be accepted until the dead-cat deadline). Your score will be recorded accordingly.
8. Your code will be awarded points only if it executes in under 10 seconds. **(Hint: Easily achievable in $O(\sqrt{n})$ time complexity!)**
9. **Modifying the website by changing its HTML to alter the submission screenshot is strictly prohibited and will be considered academic dishonesty. Such actions will be thoroughly investigated.**