# HW03 Solution

# Prob 2: $L = \{a^i : i \neq 1000\}$. DFA

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Make 0-state of Mod-11 loop a final state.
This final state accepts $a^i$ iff $(\exists x, y \in \mathbb{N})[i = 10x + 11y + 11]$ iff $i \geq 101$.

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NFA has $1 + 11 + 11 + 2 + 3 + 5 + 7 = 40$ states.

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**YES**- and this is known

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Is there a shorter regex? NO.

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**Convention** We take $\lceil \lg(x) \rceil$ to be the length of the binary number $x$. This is not quite right but we don't care. The main point is that Textbook Regex's are much shorter. We leave off the ceiling sign.

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$$\{a\} \cup \{aa\} \cup \cdots \cup \{aa \cdots a\} \cup a \cdots aa^*$$

(The second $\cdots$ is 99 $a$'s. The third is 101 $a$'s.)

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Use Chicken McNugget Theorem++ with $13, 9$ to get
100 CANNOT be written as $13x + 9y + 5$.
Any $i \geq 101$ CAN be written as $13x + 9y + 5$.

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Any $i \geq 101$ CAN be written as $13x + 9y + 5$.

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Regex Length: $5 + 9 + 13 = 27$.
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Mod-7: $\{a^i : i \not\equiv 2 \pmod 7\}$ is
$\{e, a, aaa, aaaa, aaaaa, aaaaaa\}(aaaaaaa)^*$: 28.
Total Length: 27+3+7+16+28=81.

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# Prob 5: Regex for. . ..

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Want regex for $L$. How can I obtain one?

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1. Create a DFA $M$ for $L$. It will be easy and have $102 \times 91 = 9282$.

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2. Use the $R(i, j, k)$ construction on DFA $M$.