HW04 Solution

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$$\{e\} \cup \{a^2\} \cup \{a^4\} \cup \cdots \cup \{a^{2000}\}.$$

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DFA on Next Page

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DFA:

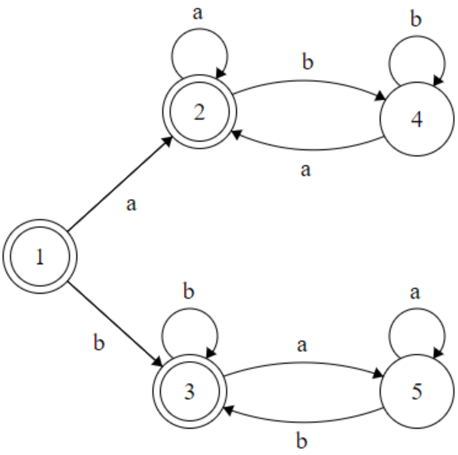

Figure: *DFA for L_4*

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Prob 3b: $L_6 = \{w : 3\#_a(w) = 3\#_b(w)\}$

Let $a^{3n}b^{2n}$ be a long string in L_6.

From this point on the proof is very similar to Part a.

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Solution on next slide.

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$\delta(p, \sigma) = \delta(p, \sigma)$.
$\delta(p, e) = \{q : (\exists \sigma \in \Sigma)[\delta(p, \sigma) = q]\}$.