HW07 Solution

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Prove **PROMISE-SAT** in P $\rightarrow$ SAT in P.

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   - If output is NO then ϕ has Hence ϕ has 1 satisfying assignments. Its. $(T, \ldots, T)$. So $\psi \notin \mathrm{SAT}$. So output NO.

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Think About What if we are promised that ϕ has either 0 or 1 satisfying assignments? If **PROMISE-SAT-0-1** problem is in P, then do you get **SAT** in P?

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So there is a fast randomized algorithm for **SAT** with a very small prob of error.

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Algorithm takes time $O((p(|x|) + q(p(|x|))$ which is poly in $|x|$.

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   Algorithm Sketch Given G, all vertices of degree $\geq k+1$ are in the VC. Remove them to form G'. Now want VC of G' of size $\leq k'$. If G' has a VC of size $\leq k'$ then G' has $\leq kk' \leq k^2$ vertices. Do Brute Force on G'.

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Abbreviated to **Respect Lower Bounds!**

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2. Bill will tell you about some kind of complexity theory to show that it is likely DS_{1000} requires $\Omega(n^{1000})$ time.
3. Bill will tell you that the the theory community has no consensus on whether DS_{1000} can be done in $n^{<1000}$.

DS_{1000} in Time n^{1000}: We Prob Can't Do Better!

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5. Hence we think $\mathrm{DS}_k \notin \mathrm{FPT}$.

3-Col and 4-col

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$$\mathrm{3COL} = \{G : G \text{ is 3-colorable }\}.$$

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$G' \in 4\text{COL}$ implies $G \in 3\text{COL}$: If $G' \in 4\text{COL}$ then coloring must use a coloring of v that is not used on any other vertex. Remove vertex v and you have a 3-coloring of G.
Thus, $G \in 3\text{COL}$ iff $G' \in 4\text{COL}$.

3-Col and 4-col

Think About Is the following true:

$$4\mathrm{COL} \leq 3\mathrm{COL}$$

3-Col and 4-col

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1. Yes, $4\mathrm{COL} \le 3\mathrm{COL}$ but the reduction is **insane**.

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4. The question of whether $4\mathrm{COL} \le 3\mathrm{COL}$ is **Unknown to Bill**

An Insane Reduction

We show that

$$4\mathrm{COL} \le 3\mathrm{COL} \text{ by an insane reduction.}$$

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I call this reduction **insane** since it goes from a graph to a formula (using Turing Machines) and then back to a graph.

Is there a Sane Reduction?

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In 2014 my students one of my students was depressed at how insane the reduction was. SO I came up with a **sane** reduction. Its on arXiv here: `https://arxiv.org/pdf/1407.5128.pdf`

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Def A graph is **planar** if it can be drawn in the plane without crossing.

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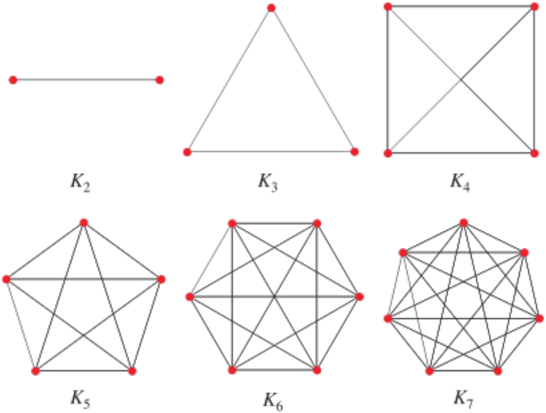

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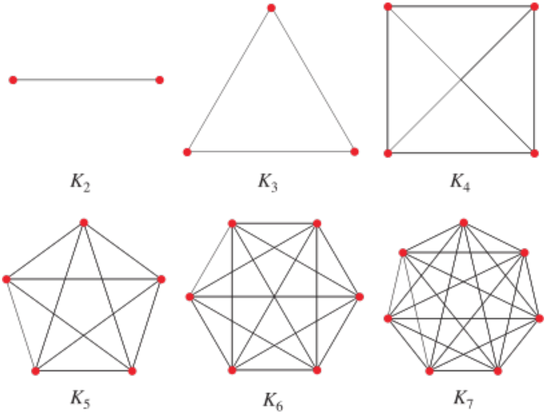

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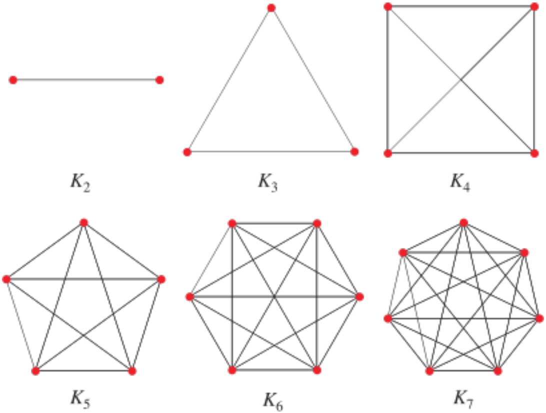

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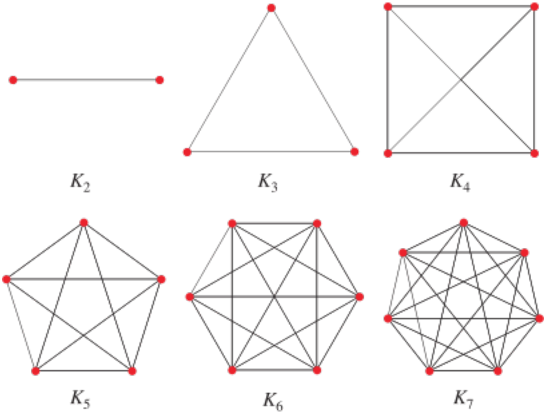

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It is known that testing if a graph is planar is in P.

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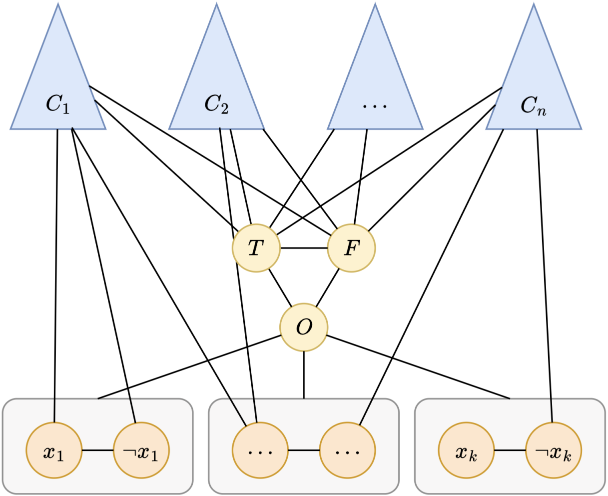

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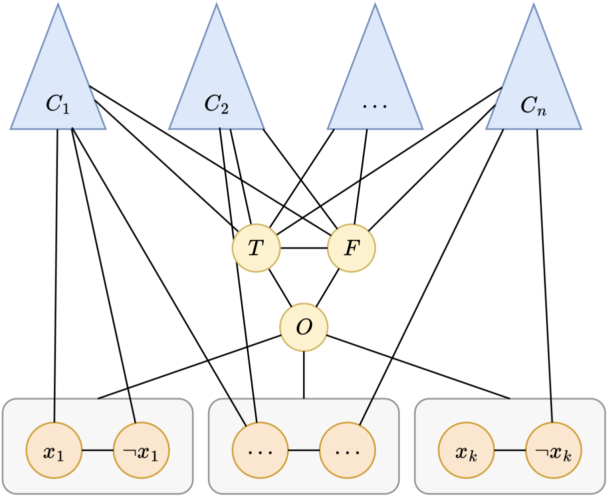

Note that the gadget is not planar.

Vote on if Planar 3COL is NP-complete

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   Fire and **Brimstone** Speech on lower bounds to follow.

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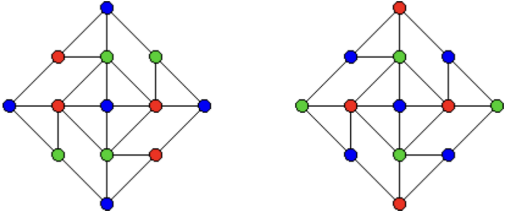

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Fire and **Brimstone** Speech on next slides.

A New Variant of Fire and Brimstone

My past **Fire** and **Brimstone** sermons:

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The bulk of the proof was the program.

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2. Hard Math

Fire and Brimstone

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So proving $\mathrm{SAT} \notin \mathrm{P}$ is going to be hard.

It is my hope that Adam-Isaac-Sam show $\mathrm{SAT} \notin \mathrm{P}$ before the final and hence prove me a fool.

Fire and Brimstone

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1. The don't have to grade the final.
2. They owe me 1000,000 free lunches.