Substitution-Permutation Networks (SPNs)

Recall...

- Want keyed permutation

$$F : \{0,1\}^n \times \{0,1\}^\ell \to \{0,1\}^\ell$$

  n = key length, ℓ = block length

- Want F_k (for uniform, unknown key k) to be indistinguishable from a uniform permutation over $\{0,1\}^\ell$

Designing block ciphers

- If x and x' differ in one bit, what should the relation between $F_k(x)$ and $F_k(x')$
  - How many bits should change (on average)?

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  - Which bits should change?

Designing block ciphers

- If x and x' differ in one bit, what should the relation between $F_k(x)$ and $F_k(x')$
  - How many bits should change (on average)? $n/2$
  - Which bits should change? unpredictable
- How to achieve this?

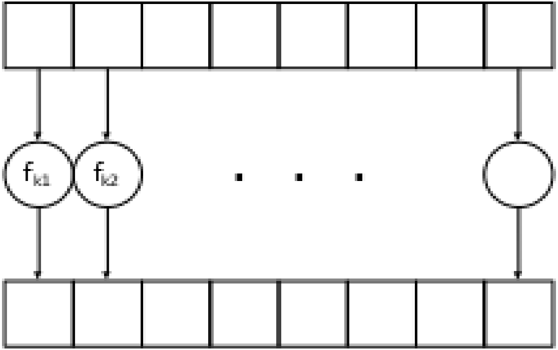
Confusion/Diffusion

- Confusion
  - Small change in input should result in local, “random” change in output
- Diffusion
  - Local change in output should be propagated to entire output

Substitution-Permutation Networks (SPNs)

- Build random-looking perm on large input from rand perms on small inputs
- E.g. assume 8-byte block length
- $$F_k(x) = f_{k1}(x_1) f_{k2}(x_2) \dots f_{k8}(x_8)$$
  where each f_{ki} is a random permutation of $n/8$ numbers.
- Need k to code 8 perms of $n/8$ numbers. Clunky.
  Need the perms to be fast AND random-looking. Hard!
  Punchline: Won't be using this but pretend for now to see what we aspire to.

Substitution-Permutation Networks (SPNs)



Is this a pseudorandom function? Vote

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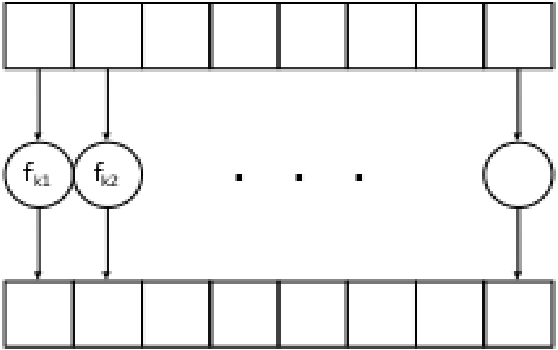
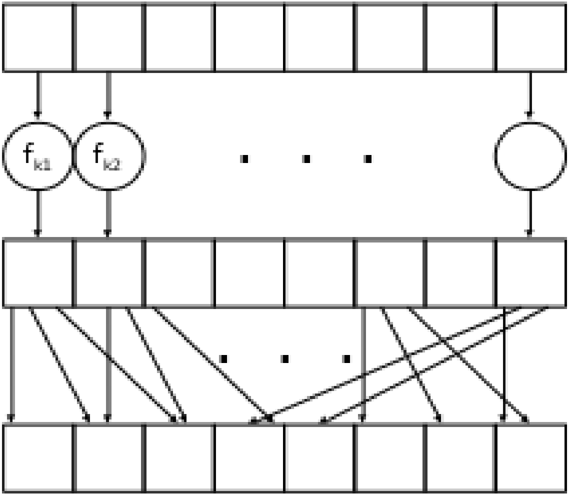


Is this a pseudorandom function? Vote No Too Local

SPN

- This has confusion but no diffusion. Random-looking locally but not globally.
  - Add a *mixing permutation*. . .

SPN



Invertibility

- Note that the structure is invertible (given the key) since the fs are permutations

SPN

- Mixing permutation is public
  - Chosen to ensure good diffusion
- Does this give a pseudorandom function?
- What if we repeat for another round (with independent, random functions)?
  - What is the minimal # of rounds we need?
  - Avalanche effect: Small change in input leads to global change.

SPN

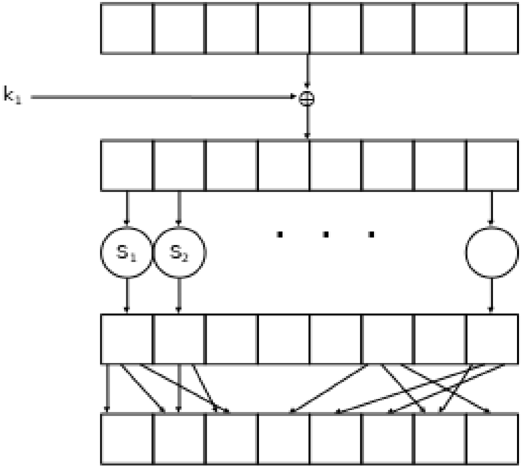
1. From key k get 8 random perms on $n/8$ bit
2. $F_k(x) = f_{k1}(x_1) \cdots f_{k8}(x_8)$ where $x = x_1 \cdots x_8$.
3. Permute the blocks.
4. Lather, Rinse, Repeat many times.

PRO: Provably gives pseudorandom perm
CON: Hard to generate fast random perms.

SPN

Key will not code perms. Key will be $k = k_1 \cdots k_{n/8}$ and k_i's will be used along with public S-box to create perms.

- $f_{k_i}(x) = S_i(k_i \oplus x)$, where S_i is a public permutation
- S_i are called "S-boxes" (substitution boxes)
- XORing the key is called "key mixing"
- Note that this is still invertible (given the key)


k_1
S_1
S_2

Avalanche effect

- Design S-boxes and mixing permutation to ensure avalanche effect
  - Small differences should eventually propagate to entire output
- S-boxes: 1-bit input change $\implies \geq$ 2-bit output change
- Mixing permutation
  - Each bit output from a given S-box should feed into a *different* S-box in the next round

S-Boxes are HARD to Create

Building them is a major challenge.

Titles of Papers that tried:

The Design of S-Boxes by Simulated Annealing

A New Chaotic Substitution Box Design for Block ciphers

Perfect Nonlinear S-Boxes

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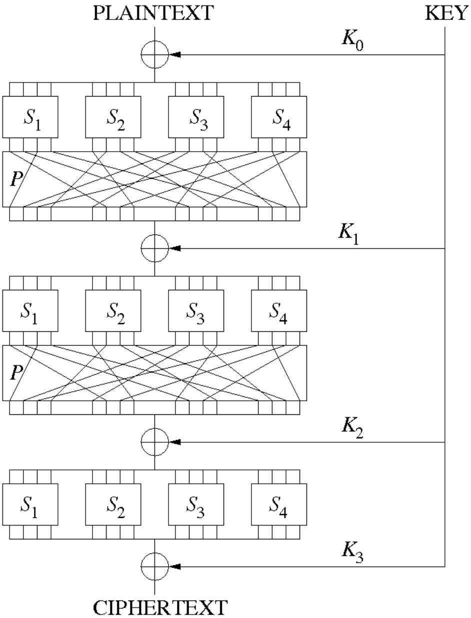
20,000. Given repeats and conference-Journal repeats, there are approx 10,000 papers on S-boxes.


PLAINTEXT
KEY
K_0
S_1
S_2
S_3
S_4
P
K_1
S_1
S_2
S_3
S_4
P
K_2
S_1
S_2
S_3
S_4
K_3
CIPHERTEXT

SPN

- One round of an SPN involves
  - Key mixing
    - Ideally, round keys are independent
    - In practice, derived from a master key via *key schedule*
  - Substitution (S-boxes)
  - Permutation (mixing permutation)
- r-round SPN has r rounds as above, plus a final key-mixing step
  - Why?
- Since S-boxes and Perms are invertible and public, if there was no final key-mixing stage then the last stage would be pointless.

Key-Recovery Attacks

- Key-recovery attacks are even more damaging than distinguishing attacks
  - As before, a cipher is secure only if the best key-recovery attack takes time $\approx 2^n$
  - A fast key-recovery attack represents a complete break of the cipher

Key-recovery attack, 1-round SPN

Consider case where there is no final key-mixing step.

1. Public input x_1
2. Then get $x_2 = k \oplus x_1$ where k is private
3. Then get x_2 and do S-box stuff to it, and Perm to it, to get x_3
4. Output x_3. Public.

If see all of this then Eve knows x_1, x_3. Can she find k? Discuss

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Yes
1) From x_3 can find x_2 since S-box stuff and Perm are all invertible.
2) Compute $x_1 \oplus x_2 = x_1 \oplus x_1 \oplus k = k$

Key-recovery attack, 1-round SPN

There is a final key-mixing step. Key $k_1, k_2 \in \{0,1\}^{n/2}$.

1. Public input x_1
2. $x_2 = k_1 \oplus x_1$ where k_1 is private
3. x_2 and do S-box stuff to it, and Perm to it, to get x_3
4. Output $x_4 = x_3 \oplus k_2$ where k_2 is private.

Eve sees x_1, x_4.
For each (k_1, k_2) see if x_1, x_4 is consistent with it. There may be many candidates. As Eve sees more input-output pairs she can zero in on the right candidate with roughly 2^n input-output pairs.
Can Eve do better?
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For each $k_2 \in \{0,1\}^{n/2}$ view the SPN as in prior slide- no last key-mixing stage. Hence can derive k_1. Have only $2^{n/2}$ candidates. Eve needs only $2^{n/2}$ input-output pairs.

Key-recovery attack, 1-round SPN, Better Attack

Work S-box-by-S-box. Assume $\frac{n}{a}$ a-to-a S-boxes.
Each guess of the first a bits of k_2 determines some a bits of k_1.
So have 2^a possibilites for $2a$-bits
Do this for first, second, ..., $\frac{n}{a}$ part of k_2
This took time

$$\underbrace{2^a + 2^a + \cdots + 2^a}_{\frac{n}{a} \text{ times}} = \frac{n2^a}{a}$$

steps. Still have $2^{n/2}$ possibilites for the key but took less time to find them.

Given an input-output pair it will likely eliminate many of the

r rounds

1) Can extend to r rounds but time complexity goes up.

2) Better than naive but still too slow.

3) Considered secure if r is large enough.

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