Key Exchange With Matrices and Lattices

October 17, 2019

DH and RSA Rely on Number Theory

1. DH and RSA rely on problems in Number Theory being hard.
2. If DL is easy then DH is cracked (not conversely).
3. If Factoring is easy then RSA is cracked (not conversely).
4. DL and Factoring are in Quantum-P (BQP).
5. If Quantum Computers (QC) ever become a reality then DH and RSA are cracked!

How worried should we be? Discuss

Is QC Really a Threat?

My opinion

1. QCs seem hard to build.
2. Recent results by Google show that QC can do some things that classical computers cannot; however, this is a long way from factoring being easy.
3. I do not work in QC or Crypto; I have no special insights.
4. QC is worth studying for the insight it gives into both quantum and computing.
5. There are classical algorithms for DL and factoring that are forcing crypto people to up their game.

Is QC Really a Threat?

My opinion

1. QCs seem hard to build.
2. Recent results by Google show that QC can do some things that classical computers cannot; however, this is a long way from factoring being easy.
3. I do not work in QC or Crypto; I have no special insights.
4. QC is worth studying for the insight it gives into both quantum and computing.
5. There are classical algorithms for DL and factoring that are forcing crypto people to up their game.

Final Opinion: Studying public-key crypto that does not depend on number theory assumptions is intellectually awesome. Might not be needed for QC, but perhaps for other scenarios.

Post-Quantum Cryptography

Assumes that Quantum Computing is Real. Consequences:

1. Factoring and DL are now easy. So can't use DH or RSA.
2. Reductions.
   - Standard Crypto: If BLAH is crackable then problem X is now easy. Easy means R (Poly time allowing coin flips and small prob of error). Used for a problem X we think is not in R.
   - Post-Quantum Crypto: If BLAH is crackable then problem X is now easy. Easy means BQP (Quantum P). Used for a problem X we think is not in BQP.

Whats True and Whats Educational

True There is a Cryptosystem based on linear algebra that is post-quantum.

Whats True and Whats Educational

True There is a Cryptosystem based on linear algebra that is post-quantum. It's secure but complicated.

Whats True and Whats Educational

True There is a Cryptosystem based on linear algebra that is post-quantum. It's secure but complicated.

Educational I came up with a similar one. It's simple but insecure.

Whats True and Whats Educational

True There is a Cryptosystem based on linear algebra that is post-quantum. It's secure but complicated.

Educational I came up with a similar one. It's simple but insecure.

Plan: I will teach the simple and insecure system that captures some of the ideas of the one that are complicated by insecure.

Whats True and Whats Educational

True There is a Cryptosystem based on linear algebra that is post-quantum. It's secure but complicated.

Educational I came up with a similar one. It's simple but insecure.

Plan: I will teach the simple and insecure system that captures some of the ideas of the one that are complicated by insecure.

Will then discuss the secure-but-complicated systems.

Terminology

The secure-but-complicated cryptosystem is called

Learning With Errors—Key Exchange. Due to Regev

Terminology

The secure-but-complicated cryptosystem is called

Learning With Errors—Key Exchange. Due to Regev

abbreviated

LWE-KE

Terminology

The secure-but-complicated cryptosystem is called

Learning With Errors—Key Exchange. Due to Regev

abbreviated

LWE-KE

The insecure-but-simple cryptosystem is called

Learning With Gasarch—Key Exchange

Terminology

The secure-but-complicated cryptosystem is called

Learning With Errors—Key Exchange. Due to Regev

abbreviated

LWE-KE

The insecure-but-simple cryptosystem is called

Learning With Gasarch—Key Exchange

abbreviated

LWG-KE

LWG-KE

October 17, 2019

LWG-KE. Two Security Parameters L, S

1. Alice generates rand prime p of length L, rand $S \times S$ matrix A over $\mathbb{Z}_p$.
2. Alice sends $(p, A, SOTE)$. All public.
3. Alice generates rand $\vec{y} \in \mathbb{Z}_p^S$. Sends $\vec{y}A$.
4. Bob geneate rand $\vec{x} \in \mathbb{Z}_p^S$, Sends $A\vec{x}$.
5. Alice computes $\vec{y}(A\vec{x}) = \vec{y}A\vec{x}$.
6. Bob computes $(\vec{y}A)\vec{x} = \vec{y}A\vec{x}$.
7. Alice and Bob have shared secret $\vec{y}A\vec{x}$

LWG-KE. Two Security Parameters L, S

1. Alice generates rand prime p of length L, rand $S \times S$ matrix A over $\mathbb{Z}_p$.
2. Alice sends $(p, A, SOTE)$. All public.
3. Alice generates rand $\vec{y} \in \mathbb{Z}_p^S$. Sends $\vec{y}A$.
4. Bob geneate rand $\vec{x} \in \mathbb{Z}_p^S$, Sends $A\vec{x}$.
5. Alice computes $\vec{y}(A\vec{x}) = \vec{y}A\vec{x}$.
6. Bob computes $(\vec{y}A)\vec{x} = \vec{y}A\vec{x}$.
7. Alice and Bob have shared secret $\vec{y}A\vec{x}$

Secure? On HW you will show that it is not secure.

LWE-KE

October 17, 2019

Small Vectors

Definition

Assume $n \in \mathbb{N}$ and p is a prime. Pick a random small $\vec{e} \in \mathbb{Z}_p^L$ means pick each component as a discrete Gaussian with mean 0 and small variance to be specified.

View $\mathbb{Z}_p$ as

$$\left\{-\frac{p-1}{2}, -\frac{p-3}{2}, \ldots, -1, 0, 1, \ldots \frac{p-3}{2}, \frac{p-1}{2}\right\}$$

Still do math mod p, but now it looks more Gaussian.

LGE-KE. Two Security Parameters L, S

1. Alice generates rand prime p of length L, rand $S \times S$ matrix A over $\mathbb{Z}_p$.
2. Alice generates rand small $\vec{y} \in \mathbb{Z}_p^S$, rand small $\vec{e}_y \in \mathbb{Z}_p^S$. Sends $\vec{y}A + 2\vec{e}_y$.
3. Bob generates rand small $\vec{x} \in \mathbb{Z}_p^S$, rand small $\vec{e}_x \in \mathbb{Z}_p^S$. Sends $A\vec{x} + 2\vec{e}_x$.
4. Alice computes $a = \vec{y}(A\vec{x} + 2\vec{e}_x) = \vec{y}A\vec{x} + 2\vec{y} \cdot \vec{e}_x$.
5. Bob computes $b = (\vec{y}A + 2\vec{e}_y)\vec{x} = \vec{y}A\vec{x} + 2\vec{x} \cdot \vec{e}_y$.
6. (This is not true, alas). Alice and Bob both take what the computed mod 2 to both get $yA\vec{x} \pmod 2$. So they both share a bit.

What they actually do is more complicated. For one thing, they only agree on the bit with high probability.

LWE-KE. Hardness Assumption

Definition

LWE (Learning with Errors) problem p a prime, $S \in \mathbb{N}$. $\vec{u} \in \mathbb{Z}_p^S$ is unknown. We want to learn $\vec{u}$. Our only operation is to

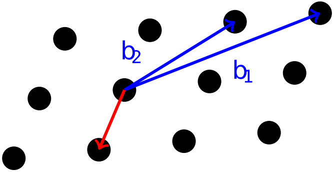
1. Pick a random $\vec{v} \in \mathbb{Z}_p^S$ small
2. Pick a random $e \in \mathbb{Z}_p$, small (you do not get to see e)
3. We get to ask for $(\vec{v}, \vec{v} \cdot \vec{u} + e)$

Solving LWE quickly means learning $\vec{u}$ with high prob after a poly (in S) number of operations.

Known If LWE-KE is crackable then LWE is easy.
So Need to have a reason why LWE is hard.
Want A Hard Problem SVP such that LWE easy implies SVP easy.

Shortest Vector Problem (SVP)

SVP Given a lattice, find the shortest Vector out of the origin.

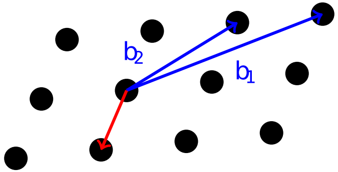


(Picture by Sebastian Schmittner - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=44488873)

Hardness Known to be NP-hard under randomized reductions.

Shortest Vector Problem (SVP)

SVP Given a lattice, find the shortest Vector out of the origin.



(Picture by Sebastian Schmittner - Own work, CC BY-SA 4.0, `https://commons.wikimedia.org/w/index.php?curid=44488873`)

Hardness Known to be NP-hard under randomized reductions.

Want SVP $\leq$ LWE $\leq$ LWE-KE. True, but with a Caveat.

LWE-KE. Hardness Assumption

Definition

LWE (Learning with Errors) problem p a prime, $n \in \mathbb{N}$. $\vec{u} \in \mathbb{Z}_p^L$ is unknown. We want to learn $\vec{u}$.

Known: If can crack LWE-KE then can solve LWE.

Known: If can solve LWE then can solve SVP problem.

Upshot: If can crack LWE-KE then can solve SVP problem.

Caveat: The sense of can solve is odd–next slides.

LWE-KE. Hardness Assumption – A Caveat

We claimed:

$$\text{SVP} \;\leq\; \text{LWE} \;\leq\; \text{LWE-KE}$$

LWE-KE. Hardness Assumption – A Caveat

We claimed:

$$\text{SVP} \;\leq\; \text{LWE} \;\leq\; \text{LWE-KE}$$

This is true. Sort of.

LWE-KE. Hardness Assumption – A Caveat

We claimed:

$$\text{SVP} \;\leq\; \text{LWE} \;\leq\; \text{LWE-KE}$$

This is true. Sort of.
It uses Quantum Reductions.

LWE-KE. Hardness Assumption – A Caveat

We claimed:

$$\text{SVP} \leq \text{LWE} \leq \text{LWE-KE}$$

This is true. Sort of.
It uses Quantum Reductions.
Recent Result Can replace Quantum with Randomized, as of last week.

LWE-KE. Hardness Assumption – A Caveat

We claimed:

$$\text{SVP} \le \text{LWE} \le \text{LWE-KE}$$

This is true. Sort of.
It uses Quantum Reductions.
Recent Result Can replace Quantum with Randomized, as of last week.
Not Quite Actually I just learned it last week, it was known for two years, but the NSA only released it last week.

LWE-KE. Hardness Assumption – A Caveat

We claimed:

$$\text{SVP} \leq \text{LWE} \leq \text{LWE-KE}$$

This is true. Sort of.
It uses Quantum Reductions.
Recent Result Can replace Quantum with Randomized, as of last week.
Not Quite Actually I just learned it last week, it was known for two years, but the NSA only released it last week.
I'm kidding

Upshot

1. QC: DH cracked, LWE-KE uncrackable if GAP-SVP hard.
2. $\neg QC$: DH looks save, LWE-KE is classically hard.

Upshot

1. QC: DH cracked, LWE-KE uncrackable if GAP-SVP hard.
2. $\neg QC$: DH looks save, LWE-KE is classically hard.

This is why post-quantum crypto uses quantum. Post-quantum crypto assumes Quantum computers exist and are fast.

- Can't use number theory assumptions like factoring hard. :-(
- Can use quantum reductions to prove hardness results. :-)

How Important Is Public Key?

October 17, 2019

Used Everywhere

Public key is mostly used for giving out keys to be used for classical systems.
This makes the following work:

Used Everywhere

Public key is mostly used for giving out keys to be used for classical systems.
This makes the following work:

1. Amazon – Credit Cards

Used Everywhere

Public key is mostly used for giving out keys to be used for classical systems.
This makes the following work:

1. Amazon – Credit Cards
2. Ebay – Paypal

Used Everywhere

Public key is mostly used for giving out keys to be used for classical systems.
This makes the following work:

1. Amazon – Credit Cards
2. Ebay – Paypal
3. Facebook privacy –

Used Everywhere

Public key is mostly used for giving out keys to be used for classical systems.
This makes the following work:

1. Amazon – Credit Cards
2. Ebay – Paypal
3. Facebook privacy – just kidding, Facebook has no privacy.

Used Everywhere

Public key is mostly used for giving out keys to be used for classical systems.
This makes the following work:

1. Amazon – Credit Cards
2. Ebay – Paypal
3. Facebook privacy – just kidding, Facebook has no privacy.
4. Every financial institution in the world.

Used Everywhere

Public key is mostly used for giving out keys to be used for classical systems.
This makes the following work:

1. Amazon – Credit Cards
2. Ebay – Paypal
3. Facebook privacy – just kidding, Facebook has no privacy.
4. Every financial institution in the world.
5. Military – though less is known about this.

Turing Awards

The Turing Award is The Nobel Prize of Computer Science.

Given out every year.

We note when someone mentioned in Public Key Crypto won.

1. 1976- Michael Rabin
2. 1995- Manuel Blum
3. 2002- Ron Rivest, Shamir, Len Adelman (RSA)
4. 2012- Silvio Micali, Shaffi Goldwasser
5. 2015- Whitfield Diffie, Martin Helman

Future: Oded Regev? Jon Katz? Ben-Brandon-Blum? Natalie-Natalie-Maddy?