A Tale of Two Books

William Gasarch

November 25, 2019

Problems with a Point: Exploring Math and Computer Science

William Gasarch
Clyde Kruskal

Problems with a Point

Ever notice how civilians (that is non-math people) use math words badly? Ever notice how sometimes you know a math statement is false (or not known) since if it was true you would know it?

Each chapter of this book makes a point like those above and then illustrates the point by doing some real mathematics.

This book gives readers valuable information about how mathematics and theoretical computer science work, while teaching them some actual mathematics and computer science through examples and exercises. Much of the mathematics could be understood by a bright high school student. The points made can be understood by anyone with an interest in math, from the bright high school student to a Field's medal winner.

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Problems with a POINT

Gasarch
Kruskal

William Gasarch • Clyde Kruskal

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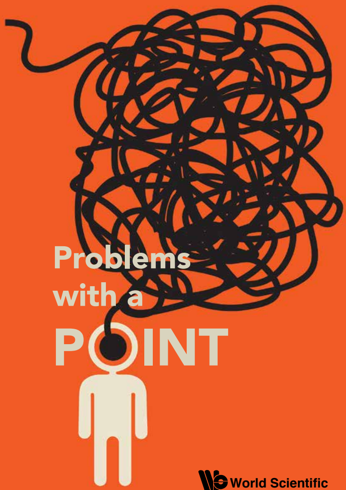

World Scientific

Book's Origin

- In 2003 Lance Fortnow started **Complexity Blog**
- In 2007 Bill Gasarch joined and it was a co-blog.
- In 2015 various book publishers asked us

  Can you make a book out of your blog?

- Lance declined but Bill said **YES**.

Book's Point

Bill took the posts that had the following format:

- make a point **about** mathematics
- do some math to underscore those points

and made those into chapters.

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Hence the title

Problems With a Point

Possible Subtitles

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The publisher wisely decided to be less cute and more informative:
Problems with a Point: Exploring Math and Computer Science

Clyde Joins the Project!

After some samples of Bill's writing the publisher said

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Please Procure People to Polish Prose and Proofs of Problems with a Point

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Now onto some samples of the book!

Point: Students Can Give Odd Answers: Easy

From the Year 2000 Maryland Math Competition:
There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

Work on it.

Point: Students Can Give Odd Answers: Easy

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Work on it.
ANSWER
If there are 45 different colors of paint then we are done. Assume there are ≤ 44 different colors. If all colors appear ≤ 44 times then there are $44 \times 44 = 1936 < 2000$ cans of paint, a contradiction.
Note: this was Problem 1, which is supposed to be easy and indeed 95% got it right. What about the other 5%? Next slide.

One of the Wrong Answers. Or is it?

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ANSWER

Paint cans are grey. Hence there are all the same color. Therefore there are 2000 cans that are the same color.

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- There are at least 45 cans that are different colors.

ANSWER

Paint cans are grey. Hence there are all the same color. Therefore there are 2000 cans that are the same color.

What do you think:

- Thats just stupid. 0 points.
- Question says *cans of the same color*.... The full 30 pts.
- Not only does he get 30 points, but everyone else should get 0.

Another Wrong Answers. Or is it?

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- There are at least 45 cans that are different colors.

ANSWER

If you look at a paint color really really carefully there will be differences. Hence, even if two cans seem to both be (say) RED, they are really different. Therefore there are 2000 cans of different colors.

Another Wrong Answers. Or is it?

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- There are at least 45 cans of the same color.
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What do you think:

- Thats just stupid. 0 points.
- Well... he's got a point. 30 points in fact.
- Not only does he get 30 points, but everyone else should get 0.

Point: Students Can Give Odd Answers: Hard

From the year 2007 Maryland Math Competition.

QUESTION: Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.

Point: Students Can Give Odd Answers: Hard

From the year 2007 Maryland Math Competition.

QUESTION: Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.

Note I think I was assigned to grade it since it looks like the kind of problem I would make up, even though I didn't. It was problem 5 (out of 5) and was hard. About 100 students tried it, 8 got full credit, 10 got partial credit

Funny Answers One

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ANSWER

All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2 + 2 = 5$ *if thats what my math teacher says. Math is pretty subjective anyway.*

Was Student One Serious?

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Theorem The students is not serious.
Proof Assume, by contradiction, that they are serious. Then they really think math is subjective. Hence they don't really understand math. Hence they would not have done well enough on Part I to qualify for Part II. But they took Part II. Contradiction.

Funny Answers Two

QUESTION: Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.

ANSWER

I like to think that we live in a world where points are not judged by their color, but by the content of their character. Color should be irrelevant in the the plane. To prove that there exists a group of points where only one color is acceptable is a reprehensible act of bigotry and discrimination.

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Was Student Two Serious. Yes.

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Was Student Two Serious. Yes. about **Justice!**.

Point: What is a Simple Function?

I assigned the following in Discrete Math: For each of the following sequences find a **simple function** $A(n)$ such that the sequence is $A(1), A(2), A(3), \ldots$

1. 10, -17, 24, -31, 38, -45, 52, $\cdots$
2. -1, 1, 5, 13, 29, 61, 125, $\cdots$
3. 6, 9, 14, 21, 30, 41, 54, $\cdots$

Caveat: These are NOT trick questions.
Work on it.

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1. 10, -17, 24, -31, 38, -45, 52, $\cdots$ $A(n) = (-1)^{n+1}(7n + 3)$.
2. -1, 1, 5, 13, 29, 61, 125, $\cdots$ $A(n) = 2^n - 3$.
3. 6, 9, 14, 21, 30, 41, 54, $\cdots$ $A(n) = n^2 + 5$.

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class so I went to Wikipedia. It said that* **A Simple Function is a linear combination of indicator functions of measurable sets.** *Is that what you want us to use?*

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I doubt the student knows what those terms mean
I doubt Bill knows what those terms mean.

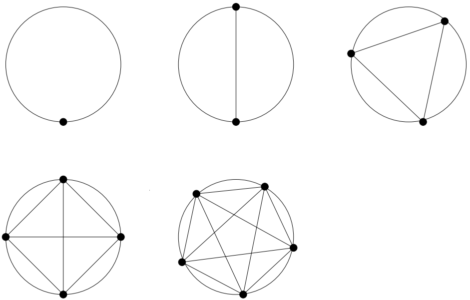
I told him NO— all Bill wanted is an easy-to-describe function. In retrospect I should have told him to use that definition to see what he came up with.
The student got the first one right, but left the other two blank.

Point: When Do Patterns Hold?

The last question brings up the question of when patterns do and don't hold. We looked for cases where a pattern *did not* hold.

First Non-Pattern: n Points on a circle

What is the max number of regions formed by connecting every pair of n points on a circle. For $n = 1, 2, 3, 4, 5$:



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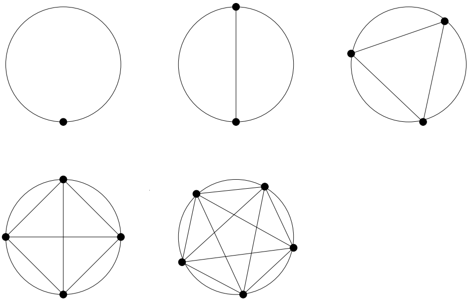

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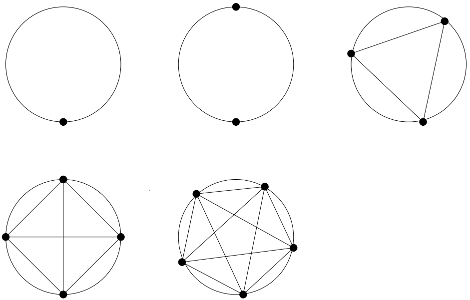

Tempted to guess 2^{n-1}.
But for $n = 6$, the number of regions is only 31.
The actual number of regions for n points is $\binom{n}{4} + \binom{n}{2} + 1$.

Second Non-Pattern: Borwein Integrals

$$\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

$$\vdots$$

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} \frac{\sin \frac{x}{5}}{\frac{x}{5}} \frac{\sin \frac{x}{7}}{\frac{x}{7}} \frac{\sin \frac{x}{9}}{\frac{x}{9}} \frac{\sin \frac{x}{11}}{\frac{x}{11}} \frac{\sin \frac{x}{13}}{\frac{x}{13}} = \frac{\pi}{2}$$

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But

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} \frac{\sin \frac{x}{5}}{\frac{x}{5}} \frac{\sin \frac{x}{7}}{\frac{x}{7}} \frac{\sin \frac{x}{9}}{\frac{x}{9}} \frac{\sin \frac{x}{11}}{\frac{x}{11}} \frac{\sin \frac{x}{13}}{\frac{x}{13}} \frac{\sin \frac{x}{15}}{\frac{x}{15}} =$$

$$\frac{467807924713440738696537864469\pi}{935615849440640907310521750000} \sim 0.99999999998529371 86 \times \frac{\pi}{2}$$

Why the breakdown at 15?

Because

$$\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{13} < 1$$

but

$$\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{15} > 1.$$

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Borwein Integral

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by William Gasarch (Author), Clyde Kruskal (Author)

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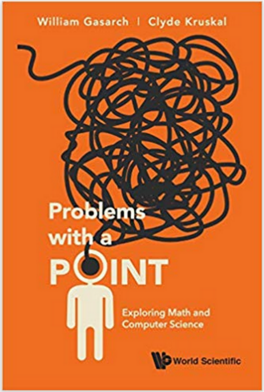

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Chapter 1 of the sequel

Problems with two Points:
More Explorations of Math and Computer Science

will be

The Mathematics of Book Pricing on Amazon

Mathematical Muffin Morsels: Nobody Wants a Small Piece

William Gasarch
Erik Metz
Jacob Prinz
Daniel Smolyak

Mathematical Muffin Morsels

Nobody wants a small piece

Suppose you have five muffins that you want to divide and give to Alice, Bob, and Carol. You want each of them to get 5/3. You could cut each muffin into 1/3-1/3-1/3 and give each student five 1/3-sized pieces. But Alice objects! She has large hands! She wants everyone to have pieces larger than 1/3.

Is there a way to divide five muffins for three students so that everyone gets 5/3, and all pieces are larger than 1/3? Spoiler alert: Yes! In fact, there is a division where the smallest piece is 5/12. Is there a better division? Spoiler alert: No.

In this book we consider THE MUFFIN PROBLEM: what is the best way to divide up m muffins for s students so that everyone gets m/s muffins, with the smallest pieces maximized. We look at both procedures for the problem and proofs that these procedures are optimal.

This problem takes us through much mathematics of interest, for example, combinatorics and optimization theory. However, the math is elementary enough for an advanced high school student.

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Vol. 16

Mathematical Muffin Morsels

Gasarch ▪ Metz
Prinz ▪ Smolyak

Problem Solving in Mathematics and Beyond
Volume 16

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World Scientific

How I Learned the Muffin Problem

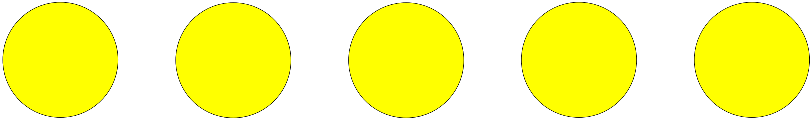
A Recreational Math Conference
(Gathering for Gardner)
May 2016

Bill found a pamphlet:

The Julia Robinson Mathematics Festival:
A Sample of Mathematical Puzzles
Compiled by Nancy Blachman

which had this problem, proposed by Alan Frank:

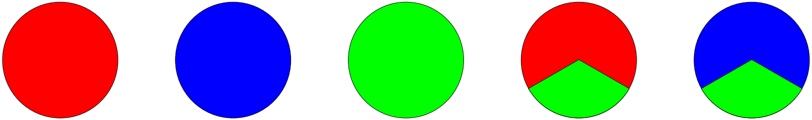
How can you divide and distribute 5 muffins to 3 students so that every student gets $\frac{5}{3}$ *where nobody gets a tiny sliver?*



Five Muffins, Three Students, Proc by Picture

Person	Color	What they Get
Alice	**RED**	$1+\frac{2}{3}=\frac{5}{3}$
Bob	**BLUE**	$1+\frac{2}{3}=\frac{5}{3}$
Carol	GREEN	$1+\frac{1}{3}+\frac{1}{3}=\frac{5}{3}$

Smallest Piece: $\frac{1}{3}$



Can We Do Better?

The smallest piece in the above solution is $\frac{1}{3}$.

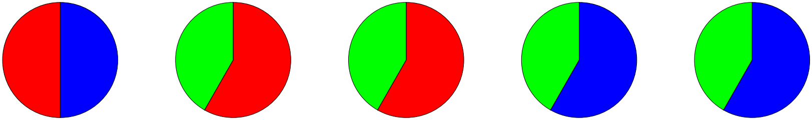
Is there a procedure with a larger smallest piece?

Work no it

Five Muffins, Three People–Proc by Picture

Person	Color	What they Get
Alice	**RED**	$\frac{6}{12}+\frac{7}{12}+\frac{7}{12}$
Bob	**BLUE**	$\frac{6}{12}+\frac{7}{12}+\frac{7}{12}$
Carol	GREEN	$\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}$

Smallest Piece: $\frac{5}{12}$



Can We Do Better?

The smallest piece in the above solution is $\frac{5}{12}$.

Is there a procedure with a larger smallest piece?

NO WE CAN'T!

5 Muffins, 3 People–Can't Do Better Than $\frac{5}{12}$

There is a procedure for 5 muffins,3 students where each student gets $\frac{5}{3}$ muffins, smallest piece N. We want $N \le \frac{5}{12}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both $\frac{1}{2}$-sized pieces to whoever got the uncut muffin. (Note $\frac{1}{2} > \frac{5}{12}$.) Reduces to other cases.
(**Henceforth:** All muffins are cut into ≥ 2 pieces.)

Case 1: Some muffin is cut into ≥ 3 pieces. Then $N \le \frac{1}{3} < \frac{5}{12}$.
(**Henceforth:** All muffins are cut into 2 pieces.)

Case 2: All muffins are cut into 2 pieces. 10 pieces, 3 students: **Someone** gets ≥ 4 pieces. He has some piece

$$\le \frac{5}{3} \times \frac{1}{4} = \frac{5}{12} \qquad \text{Great to see } \frac{5}{12}$$

General Problem

The Muffin Problem:

How can you divide and distribute m muffins to s students so that each students gets $\frac{m}{s}$ AND the MIN piece is MAXIMIZED?

This Problem went from recreational Mathematics to Serious Math when we replaced (5,3) with (m, s).

$f(m, s)$ be the smallest piece in the best procedure (best in that the smallest piece is maximized) to divide m muffins among s students so that everyone gets $\frac{m}{s}$.

We have shown $f(5, 3) = \frac{5}{12}$ here.

Amazing Results!/Amazing Theorems!

1. $f(43, 33) = \frac{91}{264}$.
2. $f(52, 11) = \frac{83}{176}$.
3. $f(35, 13) = \frac{64}{143}$.

All done by hand, no use of a computer by Co-author Erik Metz is a muffin savant !

Our First Obstacle

We solved $f(m,3)$ and $f(m,4)$ completely.
We solved $f(m,5)$ **except for $f(11,5)$**.
We had a procedure which shows $f(11,5) \geq \frac{13}{30}$.
We have an upper bound. But they don't match!

$$\frac{13}{30} \leq f(11,5) \leq \frac{11}{25} \qquad \text{Diff} = 0.006666\ldots$$

We then showed:
$f(11,5) = \frac{13}{30}$ using an **Exciting** new technique!

Terminology: Buddy

Assume that in some protocol every muffin is cut into two pieces.

Let x be a piece from muffin M.
The *other piece* from muffin M is the *buddy of* x.

Note that the buddy of x is of size

$$1 - x.$$

$f(11,5) = \frac{13}{30}$, Easy Case Based on Muffins

There is a procedure for 11 muffins, 5 students where each student gets $\frac{11}{5}$ muffins, smallest piece N. We want $N \le \frac{13}{30}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both halves to whoever got the uncut muffin. Reduces to other cases.

Case 1: Some muffin is cut into ≥ 3 pieces. $N \le \frac{1}{3} < \frac{13}{30}$.

(**Negation of Case 0 and Case 1:** All muffins cut into 2 pieces.)

$f(11,5) = \frac{13}{30}$, Easy Case Based on Students

Case 2: Some student gets ≥ 6 pieces.

$$N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.$$

Case 3: Some student gets ≤ 3 pieces.
One of the pieces is

$$\geq \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}.$$

Look at the muffin it came from to find a piece that is

$$\leq 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.$$

(**Negation of Cases 2 and 3:** Every student gets 4 or 5 pieces.)

$f(11,5)=\frac{13}{30}$, Fun Cases

Case 4: Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note ≤ 11 pieces are $> \frac{1}{2}$.

- s_4 is number of students who get 4 pieces
- s_5 is number of students who get 5 pieces

$$\begin{aligned} 4s_4 + 5s_5 &= 22 \\ s_4 + s_5 &= 5 \end{aligned}$$

$s_4 = 3$: There are 3 students who have 4 shares.
$s_5 = 2$: There are 2 students who have 5 shares.

We call a share that goes to a person who gets 4 shares a **4-share**.
We call a share that goes to a person who gets 5 shares a **5-share**.

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4.1: Some 4-share is $\le \frac{1}{2}$.
Alice gets w, x, y, z and $w \le \frac{1}{2}$.
Since $w + x + y + z = \frac{11}{5}$ and $w \le \frac{1}{2}$

$$x + y + z \ge \frac{11}{5} - \frac{1}{2} = \frac{17}{10}$$

Let x be the largest of x, y, z

$$x \ge \frac{17}{10} \times \frac{1}{3} = \frac{17}{30}$$

Look at **buddy** of x.

$$B(x) \le 1 - x = 1 - \frac{17}{30} = \frac{13}{30}$$

GREAT! This is where $\frac{13}{30}$ comes from!

$f(11,5)=\frac{13}{30}$, Fun Cases

Case 4.2: All 4-shares are $>\frac{1}{2}$. There are $4s_4=12$ 4-shares. There are ≥ 12 pieces $>\frac{1}{2}$. Can't occur.

The Future of Muffins

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But the following happened:

1. Scott Huddleston emailed me code that ALWAYS solves the problem REALLY FAST, though he had no proof of this.
2. Richard Chatwin independently discovered Scott's algorithm and prove it is correct.

Richard's paper is on arXiv.

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4. I found out from the organizers that, counting me and Alan, there should be 13 people at the talk.
5. Alan came and brought 35 muffins cut in such a way that 13 people could each get $\frac{35}{13}$ and the smallest piece was of size $\frac{64}{143}$.

Ask Bill Anything About the Books

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