BILL, RECORD LECTURE!!!!

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The One-Time Pad
Trying to Fake the OTP
Failing To Do So

The One-Time Pad

Notation Reminder: $\oplus$

Notation $\oplus$ on bits. This is often called XOR as well.

b	c	$b \oplus c$
0	0	0
0	1	1
1	0	1
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Question Why do $\wedge$, $\vee$, $\oplus$ have symbols that are commonly used but NAND and NOR do not?

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$$(\forall a, b, c \in \{0, 1\})[(a \oplus b) \oplus c = a \oplus (b \oplus c).$$

Useful Fact about $\oplus$

1. $(\forall b \in \{0,1\})[b \oplus b = 0]$
2. $(\forall b \in \{0,1\})[b \oplus 0 = b]$

Theorem $(\forall b, c \in \{0,1\})[b \oplus c \oplus c = b]$

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The Theorem is very important for the 1-time pad.

Extend $\oplus$ to Strings

Extend $\oplus$ to strings. If $x, y \in \{0,1\}^n$ then $x \oplus y$ is done bitwise.
Example $0010 \oplus 1110 = (0 \oplus 1)(0 \oplus 1)(1 \oplus 1)(0 \oplus 0) = 1100$.

1. $(\forall x \in \{0,1\}^n)[x \oplus x = 0^n]$
2. $(\forall x \in \{0,1\}^n)[x \oplus 0^n = x]$

Theorem $(\forall x, y \in \{0,1\}^n)[x \oplus y \oplus y = x]$
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- $Enc_k(m) = k \oplus m$.
- $Dec_k(c) = k \oplus c$.
- Correctness:

$$\begin{aligned} Dec_k(Enc_k(m)) &= k \oplus (k \oplus m) \\ &= (k \oplus k) \oplus m \\ &= m \end{aligned}$$

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Example Of One-Time Pad

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He sends $00111 \oplus 10100 = 10011$.

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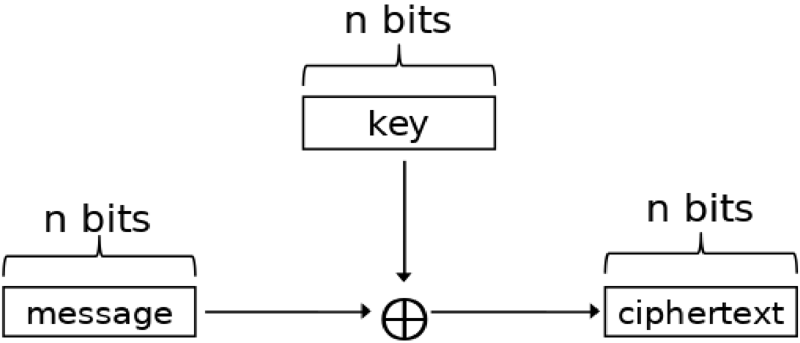
Caveat: Generating truly random bits is hard.

One-time pad



Alice and Bob Use the Psuedo One Time Pad

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- The OTP was **proven** info-theoretic secure by Shannon in 1949.

Linear Cong. Generators

How Hard is it to Generate Truly Random Bits?

Paraphrase of a **Recent Piazza conversation**

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Bill I will show what Java does and why it bytes.

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Depending on A, B, x_0 this can look random... or not.

Restrictions on A, B, M

What if M and A share a factor?

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$x_0 = 5$

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Eve will assume that A and M are rel prime. We need to assume more: next slide.

Conditions on x_0, A, B, M

1. $1 \leq x_0, A, B \leq 9999$.
2. $1000 \leq M \leq 9999$.
3. A, M are Rel Prime.

Example of Linear Cong. Gen

$x_0 = 21,\ A = 19,\ B = 30,\ M = 91$

$x_0 = 21$

$x_1 = 19 * 21 + 30 \pmod{91} = 65$

$x_2 = 19 * 65 + 30 \pmod{91} = 82$

$x_3 = 19 * 82 + 30 \pmod{91} = 41$

$x_4 = 19 * 41 + 30 \pmod{91} = 81$

$x_5 = 19 * 81 + 30 \pmod{91} = 22$

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Does this sequence look random? Hard to say.

Our Running Example

$x_0 = \mathbf{2134}$, $A = 4381$, $B = 7364$, $M = 8397$.

$$\begin{aligned} x_0 &= 2134 \text{ view as } 21, 34 \\ x_{n+1} &= 4381x_n + 7364 \pmod{8397} \end{aligned}$$

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We will assume Eve knows that the random numbers are gen by a recurrence of the form

$$x_{i+1} = Ax_i + B \pmod{M}$$

but that Eve do not know x_0, A, B, M. Does know A, M rel prime.

Alice and Bob Use the Psuedo One Time Pad

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$A = 01$, $B = 02$, $\cdots$ $Z = 26$ (**Not our usual since $A = 01$.**)
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1. Will code $m_1, m_2, \ldots$ by, **by adding mod 10 to each digit**
   Example If key is 12 38 and message is 29 23 then send

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   So send 31 51 (these do not correspond to letters, thats fine).

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How to code and decode? Next slide.

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Note E is coded as 65 and then later as 32. Recall that the whole point of OTP is that a letter won't always be coded the same way.

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The sequence is $x_0, x_1, x_2, \ldots$.
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Plaintext	$m_{1,1} m_{1,2}$	$m_{2,1} m_{2,2}$
Key	$x_{1,1} x_{1,2}$	$x_{2,1} x_{2,2}$
Alice Sends	$(m_{1,1} + x_{1,1})(m_{1,2} + x_{1,2})$	$(m_{2,1} + x_{2,1})(m_{2,2} + x_{2,2})$

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Key	$x_{1,1} x_{1,2}$	$x_{2,1} x_{2,2}$
Alice Sends	$(m_{1,1} + x_{1,1})(m_{1,2} + x_{1,2})$	$(m_{2,1} + x_{2,1})(m_{2,2} + x_{2,2})$

$(m_{1,1} + x_{1,1})(m_{1,2} + x_{1,2})$ is concatenation, not multiplication.

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Bob can keep doing this to get the entire message.

How Bob Decodes: General

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So first letter is $(c_{1,1} - x_{1,1})(c_{1,2} - x_{1,2})$.

He can keep on doing this.

Eve Can Crack The Psuedo One Time Pad

Credit Where Credit is Due

This presentation is based on the paper
Cracking a Random Number Generator by James Reed.
which is on the Course Website.

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Alice sends Bob a document using the x_i as a two chars at a time.

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Text-Digits	16	01	11	09	19	20	01	14

Thought Experiment

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Etc.

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Next slide gives complete answer.

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$$-36392598 \equiv 0 \pmod{M}$$

Can we use this?

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Hence a SMALL number of possibilities for M.

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Two ways to find possibilities for M on next few slides.

Eve Factors to Find M

Eve factors 36392598.

$$36392598 = 2 \times 3^3 \times 11 \times 197 \times 311$$

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Eve Can Crack It!–Finding M

$36392598 = 2 \times 3^3 \times 11 \times 197 \times 311$
M is a factor of 36392598 such that $7648 \leq M \leq 9999$.
How many factors of $2 \times 3^3 \times 11 \times 197 \times 311$?

Eve Can Crack It!–Finding M

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How many factors of $2 \times 3^3 \times 11 \times 197 \times 311$?
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   The next slide shows how to do it by hand. We won't go over it, but you can if you want.

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THIS SLIDE IS OPTIONAL

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1. Can't use 197 AND 311: $197 \times 311 = 61267 > 9999$.
2. If use 311 then need a 3: $2 \times 11 \times 311 = 6842 < 7648$.
3. If use 311 and exactly one 3 does not work:
   (a) Use 2 but not 11: $311 \times 3 \times 2 = 1866 < 7648$
   (b) Use 11: $\ge 311 \times 3 \times 11 = 10263 > 9999$.
4. If use 311, at least two 3's, and 11:
   $311 \times 11 \times 9 = 30789 > 9999$.
5. If use 311 and 9 does not work: $311 \times 2 \times 9 = 5598 < 7648$.
6. If use 311 and 27: $311 \times 27 = 8397$. WORKS!
7. Leave it to you to show that using 197 does not work.
8. So $M = \mathbf{8397}$.

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Two ways to find **possibilities for M**

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Two ways to find **possibilities for M**

1. Look at all 64 factors and see which ones are in $[7648, 9999]$.
2. Even less clever: Look at ALL numbers in $[7648, 9999]$ and see which ones are factors of M.

Reflect

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We might have found **several** M works. In that case, do what is on the next few slides with each one.

Eve Determines Which M Is Correct, If Any

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Multiply both sides of EQ4 by 1982 to get:

$$-6823 \times 1982 \equiv A \pmod{8397}$$

$$A \equiv -6823 \times 1982 \equiv \mathbf{4381} \pmod{8397}$$

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Upshot **If Eve's Guess Is Correct Then** $A = 4381$, $B = 7364$, $M = 8397$.

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$$x_n \equiv 8374x_{n+1} - 6965 \equiv 8374x_{n+1} + 1432$$

How will this help us?

Eve Finds x_0 (cont)

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Are we done yet? No.

Eve Uses Is-English

Eve has x_0, A, B, M so Eve can generate the **entire** key.

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If Eve's Guess Is Not Correct then either the procedure would have failed long before this point OR we find ISNOT-English.

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Most of the time she will be wrong. But the one time she is right, she will have decoded the message.

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   2.2 Use A, B, M, x_0 to generate **entire** key. Decode **entire** text. If IS-ENGLISH=YES, DONE! Else goto next L-let-seq.

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Our scenario is closer to **random** than to **Alice** .

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   3.2 They are fine for randomized algorithms (like quicksort).

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We do a very small example with a smaller word size than is used. The **Mersenne Twister** generates a sequence of 10-bit numbers (two 5-bit numbers, so for us 2 numbers in $\{01, \ldots, 26\}$).

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4. The larger the parameter which we have as 7, the longer the phrase has to be.

Mersenne Twister Example with Digits

Text-Letter	P	A	K	I	S	T	A	N	B	O
Text-Digits	16	01	11	09	19	20	01	14	02	15
Cipher-text	24	66	87	47	17	45	26	96	06	11
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Eve will guess the 7 and 5, does not know f, a, b

$$x_{n+7} = x_{n+5} \oplus f(x_n^{\text{first a digs}} x_{n+1}^{\text{last b digs}})$$

Mersenne Twister Example with Digits

Text-Letter	P	A	K	I	S	T	A	N	B	O
Text-Digits	16	01	11	09	19	20	01	14	02	15
Cipher-text	24	66	87	47	17	45	26	96	06	11
Key	18	65	76	48	08	25	25	82	04	04

Text-Letter	R	D	E	R	S	I	N	D	I	A
Text-Digits	18	04	05	18	19	09	14	04	09	01
Cipher-text	23	16	01	11	09	19	20	01	14	02
Key	95	12	04	03	90	10	16	07	15	09

Eve will guess the 7 and 5, does not know f, a, b

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$$1509 = 9010 \oplus f(0825^{\text{first a digs}}, 2528^{\text{last b digs}})$$

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Text-Letter	R	D	E	R	S	I	N	D	I	A
Text-Digits	18	04	05	18	19	09	14	04	09	01
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Text-Letter	R	D	E	R	S	I	N	D	I	A
Text-Digits	18	04	05	18	19	09	14	04	09	01
Cipher-text	23	16	01	11	09	19	20	01	14	02
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Text-Letter	R	D	E	R	S	I	N	D	I	A
Text-Digits	18	04	05	18	19	09	14	04	09	01
Cipher-text	23	16	01	11	09	19	20	01	14	02
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Can use recurrences to find f, a, b.

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Text-Digits	18	04	05	18	19	09	14	04	09	01
Cipher-text	23	16	01	11	09	19	20	01	14	02
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Can use recurrences to find f, a, b.Will need more equations and some guesswork, but crackable!

Upshot

Any pseudo-random generator that is based on recurrences is crackable.