BILL, RECORD LECTURE!!!!

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Something Wrong With All Ciphers So Far. Fix it with Randomization

September 25, 2021

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Eve knows that the city and state are the same!

What Does Eve Know?

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Alabama*, Arizona*, Arkansas, California, Colorado*, Delaware, Florida, New Georgia*, Idaho, Illinois*, Indianapolis, Iowa, Jersey, Kansas, Maryland*, Minneapolis, Minnesota, Mississippi*, Missouri, Montana, Nebraska, Nevada*, New York, Ohio, Oklahoma, Oregon, Tennessee*, Texas, Utah*, Virginia*, Virginia Beach, Wisconsin Dells, Wisconsin Rapids.

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There are 33 such cities, 22 of which still exist.
Eve's search for the spy is reduced!

Terminology

The problem of the same message leading to the same ciphertext is called

The NY,NY Problem.

How to Fix the NY,NY Problem

Problem If C is any of the ciphers discussed (except 1-time pad, Book-Vig) then Eve can tell when two messages are the same.

Discussion Is there a cipher for which Eve cannot tell this?

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Discuss Can we do this without a long key?

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Randomized Shift Key is a **function** $f : S \to S$.

1. To send message $(m_1, \ldots, m_L)$ (each m_i is a character):
   1.1 Pick random $r_1, \ldots, r_L \in S$.
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2. To decode message $((r_1; c_1), \ldots, (r_L; c_L))$:
   2.1 Find $(c_1 - f(r_1), \ldots, c_L - f(r_L))$.

Example

The key is $f(r) = 2r + 7$. Alice wants to send **NY,NY** which we interpret as **nyny.**
Need four shifts.

Pick random $r = 4$, so first shift is $2 \times 4 + 7 = 15$
Pick random $r = 10$, so second shift is $2 \times 10 + 7 = 1$
Pick random $r = 1$, so third shift is $2 \times 1 + 7 = 9$
Pick random $r = 17$, so fourth shift is $2 \times 17 + 7 = 15$

Send (4;C), (10;Z), (1;W), (17;N)

Eve will not be able to tell that is of the form XYXY.

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Question Is Randomized Shift crackable? Discuss.

Cracking Randomized Shift

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With a long text Rand Shift **is** crackable.
If N is long and Eve sees:

$$(r_1; \sigma_1)(r_2; \sigma_2) \cdots (r_N; \sigma_N).$$

View as:

1. There are only 26 possible r.
2. There are N pairs of the form (r_i, σ_i).
3. Some r appears $N/26$ times by PHP (Pigeon Hole Princ).

So have, with $L = \frac{N}{26}$:

$$(r; \sigma_{i_1}) \cdots (r; \sigma_{i_2}) \cdots \cdots (r; \sigma_{i_L})$$

Cracking Randomized Shift (cont)

So we have:

$$(r;\sigma_{i_1})\cdots(r;\sigma_{i_2})\cdots\cdots(r;\sigma_{i_L})$$

where L is large.
So $\sigma_{i_1},\ldots,\sigma_{i_L}$ are all coded by the same shift.

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Next Slide deals with this.

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We can do better. The r's are picked unif at random.

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Want that ALL r's appear LOTS of times.

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Want the prob that MOST r appear ALOT of times is large.

Chebyshev's Inequality (Advertisement)

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Chebyshev's Inequality is very important and shows up in computer science a lot!

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Hence can find, for all r, what shift r maps to.

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4. Can use the s_r's to decode entire message.

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PRO In order for Eve to crack it she needs a longer text than to crack Shift. So Alice and Bob are making Eve work harder.

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3. The terminology **NY,NY** problem and the example we gave are also due to me.
4. I am telling you this to warn you that if you are on a job interview with the NSA and you say **I learned to use the randomized shift to solve the NY,NY problem** they will not know what you are talking about.

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4. Cracking it takes a much longer text.

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