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October 25, 2021

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**Key:**  $k = (k_1, k_2, \dots, k_n)$ .

**Encrypt** (all arithmetic is mod 26)

$$\text{Enc}(m_1, m_2, \dots, m_N) =$$

$$m_1 + k_1, m_2 + k_2, \dots, m_n + k_n,$$

$$m_{n+1} + k_1, m_{n+2} + k_2, \dots, m_{n+n} + k_n,$$

...

**Decrypt** Decryption just reverses the process

# Three Kinds of Vigenère Ciphers

The following three slides give three kinds of Vig Ciphers. It is a rough way to divide up types of Vig ciphers. There will be some that are not quite in any category.

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We will be studying this type of Vig cipher today.

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This is called **The Vig-Book Cipher**. We will touch on it briefly later.

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It is usually done with alphabet  $\{0, 1\}$  or  $\{0, \dots, 9\}$ , not  $\{a, \dots, z\}$ .

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- ▶ History of Cryptography is hard since, unlike most science, people can discover things and NOT brag about it.

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**Important:** Very likely that **aiq** encrypted **the same** 3-letter sequence and hence the **length** of the key is a divisor of

$87-57=30$            $102-87=15$            $162-102=60$

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**Good Enough:** We got the key length down to a small finite set.

# Important Point About Letter Freq (I)

In an English text  $T$  of length  $N$ , where  $N$  is large:

$e$  occurs  $\sim 13\%$        $t$  occurs  $\sim 9\%$        $a$  occurs  $\sim 8\%$

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In an English text  $T$  of length  $N$ , where  $N$  is large and  $i \ll N$ , then if you take **every  $i$ th letter of  $T$** :

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Etc- have the other letters same frequencies as normal texts.



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### Sum Up Point

- ▶ If  $T$  is English or shifted-English then  $f_T \cdot f_T \sim 0.065$  large.
- ▶ If  $T$  is not English nor shifted-English then  $f_T \cdot f_T$  is prob small.

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3. If  $f_{T_0} \cdot f_{T_0}$  is small then key length probably not 10.

Next slide generalizes this.

# Testing if Key Length is $L$

**Our question**  $T$  is ciphertext coded with Vig Cipher. Eve thinks the key length is  $L$ . Let  $S$  be **every  $L$ th letter of  $T$** . SO

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- ▶ If keylength is not  $L$  then  $S$  is a ... a real mess!!  $f_S \cdot f_S$  will be small.

**Upshot** We have a test whether some text is from the shift-cipher or not. We will use it on the every- $L$ th-letter text of  $T$ .

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- ▶ Just to make sure, check another stream.

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Method TWO:

1. Let  $K = \{1, \dots, 100\}$  (I am assuming key length  $\leq 100$ ).

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**Question** Computers reduce the need for cleverness. Is this good or bad?

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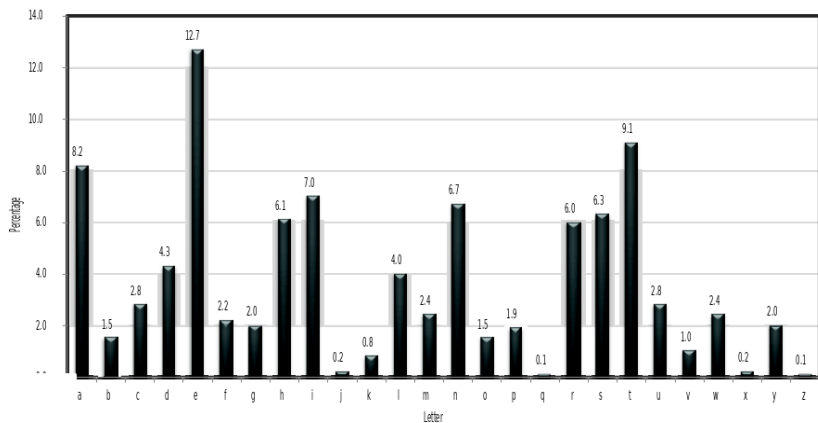
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3. You now know all shifts for all positions. Decrypt!

# Using Plaintext Letter Frequencies



# Vig-Book Cipher

October 25, 2021

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A student said:

*Let's use Vig cipher with a book for the key*

Is it a good idea? **Discuss**



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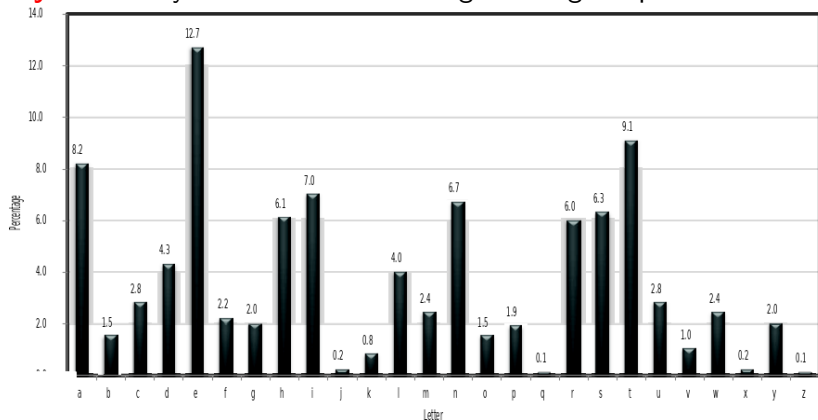
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Is it a good idea? **Discuss**

1. Before modern computer era: YES.
2. Now. NO.

# How to Crack the Vig-Book Cipher

**Key:** Both Key and Text have the English Lang Frequencies.



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Hence the following are the only possibilities for  
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6. Triples. Etc.

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Were these good choices? NO. They are books Eve might guess.

# Bill Should Not Use...

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William Gasarch • Clyde Kruskal

## Problems with a Point

Ever notice how civilians (that is non-math people) use math words badly? Ever notice how sometimes you know a math statement is false (or not known) since if it was true you would know it?

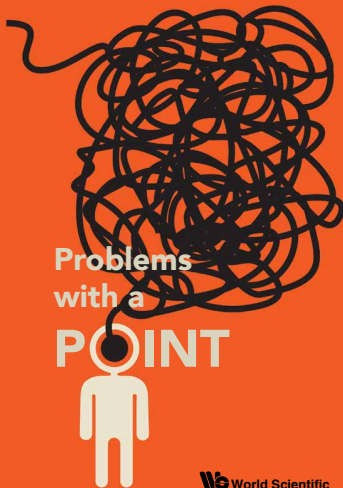
Each chapter of this book makes a point like those above and then illustrates the point by doing some real mathematics.

This book gives readers valuable information about how mathematics and theoretical computer science work, while teaching them some actual mathematics and computer science through examples and exercises. Much of the mathematics could be understood by a bright high school student. The points made can be understood by anyone with an interest in math, from the bright high school student to a Field's medal winner.

Problems with a POINT

Gasarch  
Kruskal

**World Scientific**  
www.worldscientific.com  
11261 hc



**World Scientific**

# Would make a Good Ugrad Project

Cracking the book cipher would make a good ugrad project.



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