

Final

Morally DUE Monday May 16 at 3:30PM. Dead Cat Wed May 18 at 3:30

1. (0 points) What is your name? Write it clearly.
2. (24 points) Let $R_a(k)$ be the least n such that
for all COL: $\binom{[n]}{a} \rightarrow [2]$ there exists a homog set of size k .

Assume that Zan and Not-Zan have shown that $R_3(k) \leq 2^{100k}$.

Using this to find an upper bound on $R_4(k)$ of the form $R_4(k) \leq 2^{2^{dk}}$.
Give the d and the proof.

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3. (24 points) Prove or Disprove:

For every $\text{COL}: \mathbb{Q} \rightarrow [\omega]$ *there exists an* $H \subseteq \mathbb{Q}$ *such that*

- *H has the same order type as the rationals which means all of the following hold:*
 - a) *H is countable*
 - b) *H is dense:* $(\forall x, y \in H)[x < y \implies (\exists z)[x < z < y]]$.
 - c) *H has no left endpoint:* $(\forall y \in H)(\exists x \in H)[x < y]$.
 - d) *H has no right endpoint:* $(\forall x \in H)(\exists y \in H)[x < y]$.
- *EITHER every number in H is the same color OR every number in H is a different color.*

IF you PROVE it then do a CLEAN JOB similar to the solution set on the midterm.

If you DISPROVE it then give a CLEAN counterexample.

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4. (24 points) In this problem you will prove

$$\text{PVDW}(\omega, \omega) \implies \text{PVDW}(x^3, x^3 + x^2).$$

Assume $\text{PVDW}(\omega, \omega)$ throughout this problem.

(a) (4 points) State Carefully the Lemma we need that implies

$$\text{PVDW}(x^3, x^3 + x^2).$$

(b) (10 points) Prove the Base Case of the Lemma. State carefully what from $\text{PVDW}(\omega, \omega)$ you are using.

(c) (10 points) Prove the Induction Step of the Lemma. State carefully what from $\text{PVDW}(\omega, \omega)$ you are using.

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5. (24 points) Use the Probabilistic method to get a lower bound on $W(k, 2)$ as a function of k . The function must grow faster than a polynomial in k .

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6. (4 points)

- (a) (1 points) What was your favorite theorem in this course? Why?
- (b) (1 points) What was your least theorem in this course? Why?
- (c) (2 points) Review the slides on topics I could have covered but didn't. Name a topic that I did not cover that you would have wanted me to. Why?