Homework 03

Morally Due Tue Feb 15 at 3:30PM. Dead Cat Feb 17 at 3:30

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (35 points) Give a well written complete proof of Mileti's proof of the 2-ary Can Ramsey Theorem. (I did the first two steps in class, but you will need to include those as well.)

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3. (35 points) In this problem we have a part of a proof, but want a theorem. Fill in the BLANK and the BLAH BLAH to get a theorem OF INTEREST.

   Theorem Let X be a countable infinite set of points in the plane, no three colinear. Then there exists $Y \subseteq X$, $|Y| = \infty$, such that BLANK.

   Proof Order the points in X arbitrarily, so

   $$X = \{p_1, p_2, p_3, \ldots\}.$$

   Define a coloring COL: $\binom{N}{3} \rightarrow \mathsf{R}$ via COL(i, j, k) is the area of the triangle created by p_i, p_j, p_k.

   The number of reals used is countable so we can apply Can Ramsey.

   Hence there exists $H \subseteq \mathsf{N}$, $|H| = \infty$, H is A-homog for some $A \subseteq \{1, 2, 3\}$. Look at the set of points

   $$Y = \{p_i \colon i \in H\}.$$

   Then BLAH BLAH so Y is BLANK.

   End of Proof of Theorem

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4. (30 points) **Point of this Problem** The first day of class we proved that no matter how you color the edges of K_6 there will be a monochromatic triangle. What about K_5? It turns out that there IS a coloring of K_5 with NO mono triangles. But how common is that? In this problem you will generate 1000 random 2-colorings of the edges of K_5 and COUNT how many have 0 mono triangle, 1 mono triangle, ..., 10 mono triangles. You will generate these colorings 9 different ways. Each time you do it you will count how many of the colorings had 0 mono triangles, 1 mono triangle, ..., 9 mono triangles. For $0 \le i \le 10$, n_i will be the number that have i mono triangles.

   NOTE: All we want to hand in will be the table of data, and some speculation about theorems, NOT the code itself.

   On the next page IS the problem formally.

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ONE (0 points but you need to do this for later parts) Write a program that will take input $0 \le p \le 1$ and randomly assign colors to the edges of K_5 with each edge being RED with prob p and BLUE with prob $1-p$. (You might want to use 0 and 1 instead of RED and BLUE since computers operate that way.)

TWO (0 points but you need to do it for later parts) Write a program that will, given a 2-coloring of K_5, count how many monochromatic triangles it has.

THREE (0 points but you need this for later parts) Write a program that does the following (I use psuedocode.)

For $p = 0.1, 0.2, \ldots, 0.9$

i. $n_0 = 0$, $n_1 = 0$, …, $n_{10} = 0$. (Recall that n_i will be the number of colorings that have i mono triangles. Initially this is 0.)

ii. For $i = 1$ to 1000

  A. Randomly color the edges of K_5 by coloring RED with prob p and BLUE with prob $1-p$. (You may want to use colors 0 and 1 instead.)

  B. Find j, the number of mono triangles.

  C. $n_j = n_j + 1$.

FOUR (30 points) Use your program to produce the a table of data The table should look like what is below except that (1) I made up the numbers, and (2) your table should not have any DOT DOT DOT in it, it should have all the numbers.

p	n_0	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}
0.1	0	10	10	10	10	10	10	10	10	5	5
0.2	0	10	10	10	10	10	10	10	10	7	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.9	100	0	0	0	0	0	0	0	0	0	0

FIVE (0 points) Looking at the data formulate a conjecture about colorings of K_5. Prove your conjecture.

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5. (Extra Credit) Give a well written clean proof of 3-ary Can Ramsey. There are three ways to do this. The more ways you do, the more extra credit you get!

   (a) Use some a-ary Ramsey Theorem and lots of cases (with good notation you can consolidate them), and all cases easy.

   (b) Use some a-ary Ramsey Theorem with fewer cases than the proof suggested in Part 1 (with good notation you can consolidate them), and the rainbow case will need a version of maximal sets.

   (c) Use a Mileti-style proof. Note that 2-ary Mileti used 1-ary Can Ramsey. Similarly, 3-ary Mileti will use 2-ary Can Ramsey. It will be similar to the proof of 3-ary Ramsey from 2-ary Ramsey.