Homework 05

Morally Due Tue Feb 29 at 3:30PM. Dead Cat March 2 at 3:30PM

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (30 points) Prove the following (the finite 3-hypergraph Ramsey Theorem) by using the infinite 3-hypergraph Ramsey Theorem.

   For all k, c there exists n such that for all $\mathrm{COL}\colon \binom{[n]}{3} \to [c]$ *there exists a homog set of size k.*

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3. (30 points) We will look at the following statement which we call FCR (Finite Can Ramsey)

   For all k, c *there exists* n *such that for all* $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow \omega$ *either there is a homog set of size* k *OR a min-homog set of size* k *OR a max-homog set of size* k *OR a rainbow set of size* k*.*

   (a) (10 points) TRY to prove FCR from the infinite Can Ramsey Theorem on Graphs. YOU WILL FAIL. Where does it fail?

   (b) (20 points) Prove FCR somehow (Hint: DO NOT try to finitize Mileti's proof. That can be done but is messy.)

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4. (40 points) In this problem we will look at the following problems:

   Given $X \subseteq \mathsf{R}^1$ (or S^1 or R^2) of size n show there is a large subset of size $\Omega(f(n))$ (you will figure out the f) where all of the distances are different. (S^1 is any circle.)

   We will NOT use Can Ramsey.

   Let X be a set of points (could be in R^1 or S^1 or R^2). Let $M \subseteq X$. M is *d-maximal* if (1) every pair of distances is different and (2) for all $p \in X - M$, statement (1) is false for $M \cup \{p\}$. (Note that we DO NOT have the color-degree bound we had in the past.)

   (a) (20 points) Find an increasing function f such that the following is true:

      If $X \subseteq \mathsf{R}^1$ is a set of n points then every d-maximal set is of size $\Omega(f(n))$.

   (b) (20 points) Find an increasing function g such that the following is true:

      If $X \subseteq \mathsf{S}^1$ is a set of n points then every d-maximal set is of size $\Omega(g(n))$.