Homework 08
Morally Due Tue April 5 at 3:30PM. Dead Cat April 7 at 3:30
WARNING: THE HW IS THREE PAGES LONG

1. (0 points) What is your name? Write it clearly. When is the take-home final due?

2. (35 points) Give a sentence ϕ in the language of graphs such that

$$\mathrm{spec}(\phi) = \{n \colon n \equiv 1 \pmod 4\}.$$

GO TO NEXT PAGE

3. (35 points) For this problem we are use the language of 3-hypergraphs. So there is only one predicate: $E(x, y, z)$. (We assume E is symmetric so

$$E(x, y, z) = E(x, z, y) = E(y, x, z) = E(y, z, x) = E(z, x, y) = E(z, y, z).$$

)

Let ϕ be a sentence in this language of the form

$$\phi = (\exists x_1) \cdots (\exists x_n)(\forall y_1) \cdots (\forall y_m)[\psi(x_1, \ldots, x_n, y_1, \ldots, y_m)]$$

Fill in the blank in the following theorem and proof the theorem. Make it VERY CLEAR what your XXX is. (The TAs get annoyed if they have to search for it. They also get annoyed when I ask them to search for $R(5)$.)

If $(\exists N \geq XXX(n, m))[N \in \text{spec}(\phi)]$ then

$$\{n + m, n + m + 1, \ldots\} \subseteq \text{spec}(\phi).$$

GO TO NEXT PAGE

4. (30 points) (This problem is inspired by my talk on my book.)

   A number of the form $x^2 + x$ where $x \in \mathsf{N}$, $x \geq 1$, is called a *Liam*. The first few Liam's are $2, 6, 12, 20, 30, 42, 56, 72, 90$.

   Let $L(c)$ be the least n (if it exists) so that for all c-colorings of $\{1, \ldots, n\}$ there exists two numbers that are the same color that are a Liam apart.

   (a) Find an upper bound on $L(2)$.

   (b) Find an upper bound on $L(3)$.