Homework 13
Morally Due Tue May 10 at 3:30PM. Dead Cat May 12 at 3:30
WARNING: THE HW IS TWO PAGES LONG

1. (0 points) What is your name? Write it clearly. When is the take-home final due?

2. (50 points) In this problem we look at the problem of dividing 8 muffins for 7 people so that everyone gets $\frac{8}{7}$. Recall that $f(8,7)$ is the size of the smallest piece in an optimal protocol.

   (a) (10 points) Use the Floor-Ceiling Formula to get an upper bound on $f(8,7)$. Express as both a fraction and in decimal up to 3 places.

   (b) (25 points) Use the HALF method to show that $f(8,7) \le \frac{5}{14}$. You can assume that each muffins is cut into 2 pieces so that there are 16 pieces. You can assume that nobody gets just 1 share (if they did then they would have 1 muffins, but they should get $\frac{8}{7} > 1$).

   (c) (15 points) Give a PROTOCOL that achieves the bound $\frac{5}{14}$. We give the format we want for the $f(5,3)$ problem. Do a similar format.

   $f(5,3) \ge \frac{5}{12}$:

      i. Divide 1 muffins $(\frac{6}{12}, \frac{6}{12})$.
      ii. Divide 4 muffins $(\frac{5}{12}, \frac{7}{12})$.
      iii. Give 2 students $\{\frac{6}{12}, \frac{7}{12}, \frac{7}{12}\}$.
      iv. Give 1 students $\{\frac{5}{12}, \frac{5}{12}, \frac{5}{12}\}$.

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3. (40 points) Reread the proof that, for all c, there exists a graph G_c such that $\chi(G_c) = c$ and $g(G_c) = 6$. Let M_c be the number of notes in G_c.

   Write a recurrence for the M_c in terms of M_{c-1}.

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4. (0 points– For your own Enlightenment) Reread the proof that, for all c, there exists a graph G_c such that $\chi(G_c) = c$ and $g(G_c) = 9$. It used PVDW and *easy number theory*.

   Write down a theorem in Number Theory whose truth would show that the construction yields $g(G_c) \geq 12$.

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5. (10 points) For each of the following say if its an APPLICATION!, an "application" or a "are you kidding me?" Explain why you think so.

   (a) Using Ramsey Theory to prove the Bolzano-Weierstrass Theorem.
   (b) Using Hypergraph Ramsey to prove Can Ramsey.
   (c) Using Ramsey to assist program checkers to prove programs halt.
   (d) Using Can Ramsey to show that for all infinite sets of points in the plane there is an infinite subset where all distances are distinct.
   (e) Using Ramsey to show that if $(X, \preceq)$ is a wqo then every sequence has an infinite increasing subsequence.
   (f) Using Ramsey to show that, for all k, there exists N, so given any N points in the plane, no 3 colinear, there is a set of k that form a convex k-gon.
   (g) Using Ramsey to show that in the language of graphs (or colored $\le a$-ary hypergraphs) the spectrum problem for $\exists^*\forall^*$ sentences is decidable.
   (h) Using Ramsey to show that if the universe is big enough then tables should be sorted.
   (i) Using Poly VDW to construct graphs G with $\chi(G) = c$ and $g(G) = 9$.
   (j) Using VDW to show there that, for all p, there exists long sequences of consecutive squares mod p (if I get to it).