1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (20 points) Let $R_a(k)$ be the least $n$ such that

for all COL: $\binom{[n]}{a} \rightarrow [2]$ there exists a homog set of size $k$.

Assume that Zan and Not-Zan have shown that $R_3(k) \leq 2^{100k}$ (they have not done this).

Using this find an upper bound on $R_4(k)$ of the form $R_4(k) \leq 2^{2^{dk}}$. Give the $d$ and the proof.
3. (20 points) Prove or Disprove:

For every COL: \(\mathbb{Q} \rightarrow \omega\) there exists an \(H \subseteq \mathbb{Q}\) such that

- \(H\) has the same order type as the rationals:
  - a) \(H\) is countable
  - b) \(H\) is dense: \((\forall x, y \in H)[x < y \implies (\exists z)[x < z < y]].\)
  - c) \(H\) has no left endpoint: \((\forall y \in H)(\exists x \in H)[x < y].\)
  - d) \(H\) has no right endpoint: \((\forall x \in H)(\exists y \in H)[x < y].\)

- EITHER every number in \(H\) is the same color OR every number in \(H\) is a different color.

IF you PROVE it then do a CLEAN JOB similar to the solution set on the midterm.

If you DISPROVE it the give a CLEAN counterexample.
4. (20 points) In this problem we guide you through a proof that

\[ \text{PVDW}(\omega, \omega) \implies \text{PVDW}(x^3). \]

Assume \( \text{PVDW}(\omega, \omega) \) throughout this problem.

(a) State Carefully the Lemma we need that implies \( \text{PVDW}(x^3) \).

(b) Prove Base Case of the Lemma. State carefully what from \( \text{PVDW}(\omega, \omega) \) you are using.

(c) Prove the Induction Step of the Lemma. State carefully what from \( \text{PVDW}(\omega, \omega) \) you are using.
5. (20 points) Prove the following (Use my Rado Thm Slides as a guide).

Let $a_1, \ldots, a_n \in \mathbb{Z}$.

Assume that $k \leq n$ and $a_1 + \cdots + a_k = 0$.

Then there exists a number $R = R(a_1, \ldots, a_n, c)$ such that, for all
COL: $[R] \to [c]$ there exists $x_1, \ldots, x_n \in \mathbb{Z}$, all the same color, such that

$$a_1 x_1 + \cdots + a_n x_n = 0.$$
6. (10 points) For each of the following say if its an APPLICAION!, an “application” or a “are you kidding me?” Explain why you think so.

(a) Using hypergraph Ramsey to prove Can Ramsey.
(b) Using Ramsey to assist program checkers to prove programs halt.
(c) Using Can Ramsey to show that for all infinite sets of points in the plane there is an infinite subset where all distances are distinct.
(d) Using Ramsey to show that if \((X, \preceq)\) is a wqo then every sequence has an infinite increasing subsequence.
(e) Using Ramsey to fool the class into thinking \(R(5)\) was known via the study of history.
(f) Using Ramsey to show that, for all \(k\), there exists \(N\), so given any \(N\) points in the plane, no 3 colinear, there is a set of \(k\) that form a convex \(k\)-gon.
(g) Using Ramsey to show that in the language of graphs (or colored \(\leq a\)-ary hypergraphs) the spectrum problem for \(\exists^*\forall^*\) sentences is decidable.
(h) Using Ramsey to show that if the universe is big enough then tables should be sorted.