Final

Morally DUE Monday May 16 at 3:30PM. Dead Cat Wed May 18 at 3:30

1. (0 points) What is your name? Write it clearly.

2. (24 points) Let $R_a(k)$ be the least n such that
   for all COL: $\binom{[n]}{a} \rightarrow [2]$ *there exists a homog set of size* k.

   Assume that Zan and Not-Zan have shown that $R_3(k) \le 2^{100k}$.

   Using this to find an upper bound on $R_4(k)$ of the form $R_4(k) \le 2^{2^{dk}}$.
   Give the d and the proof.

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3. (24 points) Prove or Disprove:

   For every COL: $\mathsf{Q}\to[\omega]$ *there exists an* $H \subseteq \mathsf{Q}$ *such that*

   - *H has the same order type as the rationals which means all of the following hold:*

     a) H is countable

     b) H is dense: $(\forall x, y \in H)[x < y \implies (\exists z)[x < z < y]$.

     c) H has no left endpoint: $(\forall y \in H)(\exists x \in H)[x < y]$.

     d) H has no right endpoint: $(\forall x \in H)(\exists y \in H)[x < y]$.

   - *EITHER every number in H is the same color OR every number in H is a different color.*

   IF you PROVE it then do a CLEAN JOB similar to the solution set on the midterm.

   If you DISPROVE it then give a CLEAN counterexample.

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4. (24 points) In this problem you will prove

$$\mathrm{PVDW}(\omega,\omega) \implies \mathrm{PVDW}(x^3, x^3+x^2).$$

Assume PVDW(ω,ω) throughout this problem.

(a) (4 points) State Carefully the Lemma we need that implies

$$\mathrm{PVDW}(x^3, x^3+x^2).$$

(b) (10 points) Prove the Base Case of the Lemma. State carefully what from PVDW(ω,ω) you are using.

(c) (10 points) Prove the Induction Step of the Lemma. State carefully what from PVDW(ω,ω) you are using.

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5. (24 points) Use the Probabilistic method to get a lower bound on $W(k,2)$ as a function of k. The function must grow faster than a polynomial in k.

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6. (4 points)

    (a) (1 points) What was your favorite theorem in this course? Why?

    (b) (1 points) What was your least theorem in this course? Why?

    (c) (2 points) Review the slides on topics I could have covered but didn't. Name a topic that I did not cover that you would have wanted me to. Why?