Homework 2

Morally Due Tue Feb 15 at 3:30PM. Dead Cat Feb 17 at 3:30

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (35 points) Look at the slides on the Can Ramsey Theorem. Look at the proof that uses 4-ary Ramsey.

   RECAP OF THAT PROOF FOR OUR PURPOSES: Given a coloring COL : $\binom{N}{2} \rightarrow [\omega]$ we created COL$'$: $\binom{N}{4} \rightarrow [16]$. We then applied 4-ary Ramsey Theory to get a homog set H (relative to COL$'$). We show that whatever color the homog set was we found an infinite subset of H that was with respect to COL either (a) homog, (b) max-homog, (c) min-homog, or (d) rainbow.

   OKAY, now for our problem.

   Look at the case where there is an infinite homog (using coloring COL$'$) H such that

   $$(\forall x_1 < x_2 < x_3 < x_4 \in H)[\mathrm{COL}(x_2, x_3) = \mathrm{COL}(x_1, x_4)].$$

   Show that H (or perhaps some infinite subset of it) is homog with respect to COL.

   (I am asking you to do one of the cases I skipped.)

GO TO NEXT PAGE

3. (35 points) Before the proof of Can Ramsey that used 3-ary Ramsey we proved the following:

   Assume X is infinite and $\mathrm{COL} : \binom{X}{2} \rightarrow [\omega]$. *Assume that, for all $x \in X$ and colors c,* $\deg_c(x) \le 1$. *Let M be a MAXIMAL rainbow set. Then M is infinite.*

   In this problem you will come up with and prove a FINITE version of this theorem.

   Fill in the function $f(n)$ and prove the following:

   Assume $|X| = n$ and $\mathrm{COL} : \binom{X}{2} \rightarrow [\omega]$. *Assume that, for all $x \in X$ and colors c,* $\deg_c(x) \le 1$. *Let M be a MAXIMAL rainbow set. Then* $|M| \ge \Omega(f(n))$.

GO TO NEXT PAGE

4. (30 points) In this problem we have a part of a proof, but want a theorem. Fill in the BLANK and the BLAH BLAH to get a theorem OF INTEREST.

   Theorem Let X be a countably infinite set of points in the plane. Then there exists $Y \subseteq X$, $|Y| = \infty$, such that BLANK.

   Proof Order the points in X arbitrarily, so

   $$X = \{p_1, p_2, p_3, \ldots\}.$$

   Define a coloring COL: $\binom{N}{2} \rightarrow \mathsf{R}$ via $\mathrm{COL}(i, j) = |p_i - p_j|$.

   The number of reals used is countable so we can apply Can Ramsey.

   Hence there exists $H \subseteq \mathsf{N}$, $|H| = \infty$, H is either homog, min-homog, max-homog, or rainbow. Look at the set of points

   $$Y = \{p_i \colon i \in H\}.$$

   Then BLAH BLAH so Y is BLANK.

   End of Proof of Theorem

GO TO NEXT PAGE

5. (Extra Credit) Give a well written clean proof of 3-ary Can Ramsey. There are three ways to do this. The more ways you do, the more extra credit you get!

   (a) Use some a-ary Ramsey Theorem and lots of cases (with good notation you can consolidate them), and all cases easy.

   (b) Use some a-ary Ramsey Theorem with fewer cases than the proof suggested in Part 1 (with good notation you can consolidate them), and the rainbow case will need a version of maximal sets.

   (c) Use a Mileti-style proof. Note that 2-ary Mileti used 1-ary Can Ramsey. Similarly, 3-ary Mileti will use 2-ary Can Ramsey. It will be similar to the proof of 3-ary Ramsey from 2-ary Ramsey.