Homework 2
Morally Due Tue Feb 15 at 3:30PM. Dead Cat Feb 17 at 3:30

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (35 points) Look at the slides on the Can Ramsey Theorem. Look at the proof that uses 4-ary Ramsey.
RECAP OF THAT PROOF FOR OUR PURPOSES: Given a coloring \( \text{COL} : \binom{\mathbb{N}}{2} \rightarrow \omega \) we created \( \text{COL}' : \binom{\mathbb{N}}{4} \rightarrow [16] \). We then applied 4-ary Ramsey Theory to get a homog set \( H \) (relative to \( \text{COL}' \)). We show that whatever color the homog set was we found an infinite subset of \( H \) that was with respect to \( \text{COL} \) either (a) homog, (b) max-homog, (c) min-homog, or (d) rainbow.
OKAY, now for our problem.
Look at the case where there is an infinite homog (using coloring \( \text{COL}' \)) \( H \) such that

\[
(\forall x_1 < x_2 < x_3 < x_4 \in H)[\text{COL}(x_2, x_3) = \text{COL}(x_1, x_4)].
\]
Show that \( H \) (or perhaps some infinite subset of it) is homog with respect to \( \text{COL} \).
(I am asking you to do one of the cases I skipped.)

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3. (35 points) Before the proof of Can Ramsey that used 3-ary Ramsey we proved the following:

Assume $X$ is infinite and $\text{COL} : (\mathcal{X}) \to [\omega]$. Assume that, for all $x \in X$ and colors $c$, $\text{deg}_c(x) \leq 1$. Let $M$ be a MAXIMAL rainbow set. Then $M$ is infinite.

In this problem you will come up with and prove a FINITE version of this theorem.

Fill in the function $f(n)$ and prove the following:

Assume $|X| = n$ and $\text{COL} : (\mathcal{X}) \to [\omega]$. Assume that, for all $x \in X$ and colors $c$, $\text{deg}_c(x) \leq 1$. Let $M$ be a MAXIMAL rainbow set. Then $|M| \geq \Omega(f(n))$.
4. (30 points) In this problem we have a part of a proof, but want a theorem. Fill in the BLANK and the BLAH BLAH to get a theorem OF INTEREST.

**Theorem** Let $X$ be a countably infinite set of points in the plane. Then there exists $Y \subseteq X$, $|Y| = \infty$, such that BLANK.

**Proof** Order the points in $X$ arbitrarily, so

$$X = \{p_1, p_2, p_3, \ldots\}.$$  

Define a coloring $\text{COL}: \binom{\mathbb{N}}{2} \to \mathbb{R}$ via $\text{COL}(i, j) = |p_i - p_j|$. 

The number of reals used is countable so we can apply Can Ramsey. Hence there exists $H \subseteq \mathbb{N}$, $|H| = \infty$, $H$ is either homog, min-homog, max-homog, or rainbow. Look at the set of points

$$Y = \{p_i : i \in H\}.$$  

Then BLAH BLAH so $Y$ is BLANK.

**End of Proof of Theorem**
5. (Extra Credit) Give a well written clean proof of 3-ary Can Ramsey. There are three ways to do this. The more ways you do, the more extra credit you get!

(a) Use some $a$-ary Ramsey Theorem and lots of cases (with good notation you can consolidate them), and all cases easy.

(b) Use some $a$-ary Ramsey Theorem with fewer cases than the proof suggested in Part 1 (with good notation you can consolidate them), and the rainbow case will need a version of maximal sets.

(c) Use a Mileti-style proof. Note that 2-ary Mileti used 1-ary Can Ramsey. Similarly, 3-ary Mileti will use 2-ary Can Ramsey. It will be similar to the proof of 3-ary Ramsey from 2-ary Ramsey.