Homework 04
Morally Due Tue Feb 22 at 3:30PM. Dead Cat Feb 24 at 3:30

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (40 points) Assume \((X, \preceq_X)\) and \((Y, \preceq_Y)\) are wqo. Consider the ordering \((X \times Y, \preceq)\) where \(\preceq\) is defined as

\[(x_1, y_1) \preceq (x_2, y_2) \text{ iff } x_1 \preceq_X x_2 \text{ AND } y_1 \preceq_Y y_2.\]

Show that \((X \times Y, \preceq)\) is a wqo.
3. (50 points) Assume \((X, \preceq)\) is a wqo. Let \(P^\text{finite}(X)\) be the set of finite subsets of \(X\). Let \(\preceq'\) be the following order on \(P^\text{finite}(X)\).

Let \(Y, Z \in P^\text{finite}(X)\).

\(Y \preceq' Z\) iff

there exists a FUNCTION \(f : Y \to Z\) such that \((\forall y \in Y)[y \preceq f(y)]\).

(a) (20 points) Prove or disprove: \((P^\text{finite}(X), \preceq')\) is a wqo.

(b) (15 points) Modify \(\preceq'\) such that the function \(f\) has to be injective (also called 1-1). Prove or disprove: \((P^\text{finite}(X), \preceq')\) is a wqo.

(c) (15 points) Modify \(\preceq'\) such that the function \(f\) has to be surjective (also called onto). Prove or disprove: \((P^\text{finite}(X), \preceq')\) is a wqo.
4. (10 points) GOTO my webpage of funny music and GOTO the section on Math Songs


- Listen to the Bolzano Weirstrauss rap- or as much of it as you can stand. Comment on it.
- Pick ANY OTHER math song AT RANDOM and listen to it. Is it better than the BW rap (hint: YES). Comment on it.
5. (Extra Credit- NOT towards your grade but towards a letter I may one
day write for you) (This will look like a prior extra credit but it’s a new
problem.)

Definition A bipartite graph is a graph with vertices \(A \cup B\) and the only
edges are between vertices of \(A\) and vertices of \(B\). \(A\) and \(B\) can be the
same set. We denote a bipartite graph with a 3-tuple \((A, B, E)\).

Notation \(K_{n,m}\) is the bipartite graph \(([n],[m],[n] \times [m])\).

Notation \(K_{N,N}\) is the bipartite graph \((N,N,N \times N)\).

Definition If \(COL\) is a \(c\)-coloring of the edges of \(K_{N,N}\) then \((H_1,H_2)\) is
a homog set if \(c\) restricted to \(H_1 \times H_2\) takes on only 1 value (I changed
the wording on this so I can generalize it later.)

RECALL In a prior extra credit problem we DISPROVED the fol-
lowing:

For every 2-coloring of the edges of \(K_{N,N}\) there exists \(H_1, H_2\) infinite
such that \((H_1,H_2)\) is a homog set.

In other words we showed the following:

There IS a 2-coloring of the edges of \(K_{N,N}\) such that there is NO \(H_1, H_2\) infinite such that \((H_1,H_2)\) is a homog set.

This inspires the following definition.

Definition Let \(d \leq c\). If \(COL\) is a \(c\)-coloring of the edges of \(K_{N,N}\) then
\((H_1,H_2)\) is a \(d\)-homog set if \(c\) restricted to \(H_1 \times H_2\) \(COL\) takes on \(\leq d\)
values.

SO to recap- we could have a 2-coloring of the edges of \(K_{N,N}\) where
there is no 1-homog set. But there is clearly a 2-homog set, namely
\((N,N)\).

And now FINALLY the problem:

For ever \(k \geq 3\) Prove or disprove: For every \(k\)-coloring of the edges of
\(K_{N,N}\) there exists \(H_1, H_2\) infinite such that \((H_1,H_2)\) is a 2-homog set.