

Homework 05

Morally Due Tue March 1 at 3:30PM. Dead Cat March 3 at 3:30PM

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?
2. (30 points) Prove the following (the finite 3-hypergraph Ramsey Theorem) by using the infinite 3-hypergraph Ramsey Theorem.

For all k, c there exists n such that for all $\text{COL}: \binom{[n]}{3} \rightarrow [c]$ there exists a homog set of size k .

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3. (30 points) We will look at the following statement which we call FCR (Finite Can Ramsey)

For all k, c there exists n such that for all $\text{COL}: \binom{[n]}{2} \rightarrow \omega$ either there is a homog set of size k OR a min-homog set of size k OR a max-homog set of size k OR a rainbow set of size k .

- (a) (10 points) TRY to prove FCR from the infinite Ramsey Theorem. YOU WILL FAIL. Where does it fail?
- (b) (20 points) Prove FCR somehow (Hint: DO NOT try to finitize Miletì's proof. That can be done but is messy.)

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4. (40 points) In this problem we will look at the following problems:

Given $X \subseteq \mathbb{R}^1$ (or S^1 or \mathbb{R}^2) of size n show there is a large subset of size $\Omega(f(n))$ (you will figure out the f) where all of the distances are different. (S^1 is any circle.)

We will NOT use Can Ramsey.

Let X be a set of points (could be in \mathbb{R}^1 or S^1 or \mathbb{R}^2). Let $M \subseteq X$. M is *d-maximal* if (1) every pair of distances is different and (2) for all $p \in X - M$, statement (1) is false for $M \cup \{p\}$. (Note that we DO NOT have the color-degree bound we had in the past.)

(a) (10 points) Find an increasing function f such that the following is true:

If $X \subseteq \mathbb{R}^1$ is a set of n points then every d-maximal set is of size $\Omega(f(n))$.

(b) (15 points) Find an increasing function g such that the following is true:

If $X \subseteq S^1$ is a set of n points then every d-maximal set is of size $\Omega(g(n))$.

(c) (15 points) Find an increasing function h such that the following is true:

If $X \subseteq \mathbb{R}^2$ is a set of n points then every d-maximal set is of size $\Omega(h(n))$.