Homework 06
Morally Due Tue March 8 at 3:30PM. Dead Cat March 10 at 3:30

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (35 points) Let $R_a(k)$ be the least $n$ such that
   
   for all COL: $\binom{[n]}{a} \rightarrow [2]$ there exists a homog set of size $k$.
   
   For this problem assume $R_2(k) \leq 2^{2k}$ (which is true).
   
   In class I sketched the beginning of the proof that $R_3(k) \leq 2^{2^{O(k)}}$.
   
   For this problem give a complete rigorous proof.
3. (35 points) Prove the following:

For all $k$ there exists $n$ such that for all $\text{COL} : (\{k, \ldots, n\}) \to \omega$ there exists either

- a LARGE homog set, or
- a LARGE rainbow set (all the numbers are colored differently).

**SOLUTION**

We will determine $n$ later.

Let $\text{COL} : (\{k, \ldots, n\}) \to \omega$

We define a coloring $\text{COL}' : (\{k, \ldots, n\}) \to [2]$: $\text{COL}'(x, y) = \begin{cases} 
\text{EQ} & \text{if } \text{COL}(x) = \text{COL}(y) \\
\text{NEQ} & \text{if } \text{COL}(x) \neq \text{COL}(y) 
\end{cases}$

Apply Ramsey’s theorem to get a homog set

$$\{h_1 < h_2 < \cdots\}$$

If its homog color EQ then these are all equal.

if its homog color NEQ then these are all NOT equal.

**END OF SOLUTION**
4. (30 points) Prove the following: For all $k$ there exists $n$ such that for all $COL : \binom{\{k,...,n\}}{2} \rightarrow [100]$ there exists an $H \subseteq [n]$ such that

- $H$ is a homog set, and
- $|H| \geq 2^{\min(H)}$. 