Homework 06

Morally Due Tue March 8 at 3:30PM. Dead Cat March 10 at 3:30

1. (0 points) What is your name? Write it clearly. When is the take-home midterm due?

2. (35 points) Let $R_a(k)$ be the least n such that

   for all COL: $\binom{[n]}{a} \rightarrow [2]$ *there exists a homog set of size* k.

   For this problem assume $R_2(k) \le 2^{2k}$ (which is true).

   In class I sketched the beginning of the proof that $R_3(k) \le 2^{2^{O(k)}}$.

   For this problem give a complete rigorous proof.

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3. (35 points) Prove the following:

For all k there exists n such that for all $COL : \binom{\{k,\ldots,n\}}{1} \rightarrow \omega$ there exists either

- *a LARGE homog set, or*
- *a LARGE rainbow set (all the numbers are colored differently).*

SOLUTION

We will determine n later.

Let $COL : \binom{\{k,\ldots,n\}}{1} \rightarrow \omega$

We define a coloring $COL' : \binom{\{k,\ldots,n\}}{2} \rightarrow [2]$:

$$COL'(x,y) = \begin{cases} EQ & \text{if } COL(x) = COL(y) \\ NEQ & \text{if } COL(x) \neq COL(y) \end{cases}$$

Apply Ramsey's theorem to get a homog set

$$\{h_1 < h_2 < \cdots\}$$

If its homog color EQ then these are all equal.

if its homog color NEQ then these are all NOT equal.

END OF SOLUTION

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4. (30 points) Prove the following: *For all k there exists n such that for all $COL : \binom{\{k,\ldots,n\}}{2} \rightarrow [100]$ there exists an $H \subseteq [n]$ such that*

   - *H is a homog set, and*
   - $|H| \geq 2^{2^{\min(H)}}$.