Homework 06

Morally Due Tue March 29 at 3:30PM. Dead Cat March 31 at 3:30

IN THIS HW WHENEVER I SAY "A SET OF POINTS IN THE PLANE" I MEAN THAT THEY HAVE NO THREE COLINEAR.

1. (0 points) What is your name? Write it clearly. When is the take-home final due?

2. (35 points) Let $N(k)$ be the least n such that for all sets of n points there is a subset of k of them that that form a convex k-gon.

   We begin a proof that $N(k)$ exists and you need to finish it.

   We show that $n = R_3(k)$ suffice. Let X be a set of $n = R_3(k)$ points in the plane. Let the points be $p_1, p_2, \ldots, p_n$.

   Color (p_i, p_j, p_j) (with $i < j < k$) RED if p_i, p_j, p_k is clockwise.

   Color (p_i, p_j, p_j) (with $i < j < k$) BLUE if p_i, p_j, p_k is counter clockwise.

   The Homogenous set of size k is a convex k-gon because FILL THIS IN.

GO TO NEXT PAGE

3. (35 points)

   Def PH(k) is the least $n \ge$ such that for all 2-colorings of $\binom{\{k,\ldots,n\}}{2}$ there exists a large homog set. WE ALSO REQUIRE $|\{k,\ldots,n\}| \ge 3$ SO THAT PH(1) IS NOT TRIVIAL. THERE ARE ALSO OTHER REASONS.

   Show that PH(1) ≤ 6.

GO TO NEXT PAGE

4. (30 points) Recall:

If $n \equiv 1 \pmod 2$ *then for any* $\mathrm{COL}\colon \binom{[n]}{2} \to [2]$ *there exists at least*

$$\frac{n^3}{24} - \frac{n^2}{4} + \frac{5n}{24}$$

monochromatic K_3*'s.*

We will vary this in two ways.

(a) (15 points) Find a function f such that the followings is true:

If $n \equiv 0 \pmod 2$ *then for any* $\mathrm{COL}\colon \binom{[n]}{2} \to [n]$ *there exists at least* $f(n)$ *monochromatic* K_3*'s.*

Prove your result.

(b) (15 points) We are interested in what happens if you have THREE colors. Do some empirical studies to try to find a function f such that the following holds:

If $\mathrm{COL}\colon \binom{[n]}{2} \to [3]$ *then there exists at least* $f(n)$ *monochromatic* K_3*'s. ($f(n)$ an be approximate. For example, if the problem was for 2-coloring then* $f(n)$ *could be* $\frac{n^3}{24}$*.)*

(HINT: Use the code you wrote for the midterm; however, only use the case of $p_1 = p_2 = p_3 = \frac{1}{3}$.)

(c) (Extra Credit, 0 points) PROVE a result along the lines of:

If n *satisfies condition YOU FILL IN and* $\mathrm{COL}\colon \binom{[n]}{2} \to [3]$ *then there exists at least* $f(n)$ *monochromatic* K_3*'s.)*

(HINT: Use the code you wrote for the midterm; however, only use the case of $p_1 = p_2 = p_3 = \frac{1}{3}$*.)*

GO TO NEXT PAGE

5. (Extra Credit, but THINK ABOUT IT. WARNING- I have not done this problem)

   Let X be an infinite set of points $p_1, p_2, p_3, \ldots$. Let $\mathrm{COL}\binom{\mathsf{N}}{3} \rightarrow \omega$ *be defined as follows:*

   $$\mathrm{COL}(i,j,k) = \textit{ the number of points inside the } (i,j,k) \textit{ triangle.}$$

   Apply the 3-ary Can Ramsey Theorem to this Coloring. NOW WHAT?

6. (Extra Credit, but THINK ABOUT IT–WARNING: the way I know how to do this is based on material you have not seen) We want to write a sentence ϕ in the language of graphs such that

   $G \models \phi$ IFF G has an even number of vertices.

   Is this possible?