**Homework 13**
**Morally Due Tue May 10 at 3:30PM. Dead Cat May 12 at 3:30**
**WARNING: THE HW IS TWO PAGES LONG**

1. (0 points) What is your name? Write it clearly. When is the take-home final due?

2. (30 points) In this problem we look at the problem of dividing 8 muffins for 7 people so that everyone gets $\frac{8}{7}$. Recall that $f(8,7)$ is the size of the smallest piece in an optimal protocol.

   (a) (5 points) Use the Floor-Ceiling Formula to get an upper bound on $f(8,7)$. Express as both a fraction and in decimal up to 3 places.

   (b) (15 points) Use the HALF method to show that $f(8,7) \le \frac{5}{14}$. You can assume that each muffins is cut into 2 pieces so that there are 16 pieces. You can assume that nobody gets just 1 share (if they did then they would have at most 1 muffin, but they should get $\frac{8}{7} > 1$).

   (c) (10 points) Give a PROTOCOL that achieves the bound $\frac{5}{14}$. We give the format we want for the $f(5,3)$ problem. Do a similar format.

   $f(5,3) \ge \frac{5}{12}$:

   i. Divide 1 muffins $(\frac{6}{12}, \frac{6}{12})$.
   ii. Divide 4 muffins $(\frac{5}{12}, \frac{7}{12})$.
   iii. Give 2 students $\{\frac{6}{12}, \frac{7}{12}, \frac{7}{12}\}$.
   iv. Give 1 students $\{\frac{5}{12}, \frac{5}{12}, \frac{5}{12}\}$.

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3. (30 points) Show that $f(33,7) \le \frac{33}{70}$ using the INT method. Follow the template below (you might want to use the LaTeX code itself). You may omit the cases where some muffin is uncut and where some muffin is divided into $\ge 3$ pieces.

Assume that there is a protocol for 33 muffins, 7 students. We look at all cases of what it can do and show that every case either has a piece $\le \frac{33}{70}$ or cannot occur.

**Case 1:** Some student gets $\ge V+1$ pieces. YOU NEED TO FIND $V$ SUCH THAT THIS LEADS TO A PIECE $\le \frac{33}{70}$.

**Case 2:** Some student gets $\le V-2$ pieces. YOU NEED TO FIND $V$ SUCH THAT THIS LEADS TO A PIECE $\le \frac{33}{70}$.

**Case 3:** Every student gets either $V-1$ or $V$ pieces.

Let $s_{V-1}$ be the number of of students who ge $V-1$ pieces.

Let $s_V$ be the number of of students who ge $V$ pieces.

FIND $s_{V-1}$ AND $s_V$

A $s_{V-1}$-piece is a piece that goes to a student who gets $s_{V-1}$ pieces.

A $s_V$-piece is a piece that goes to a student who gets $s_V$ pieces.

**Case 3.1** There exists a $s_V$-piece that is $> \alpha$. FIND SMALLEST $\alpha$ THAT MAKES HAVING A PIECE AN $s_V$-piece THAT IS $> \alpha$ IMPLIES SOME PIECE IS $\le \frac{33}{70}$.

**Case 3.2** There exists a $s_{V-1}$-piece that is $< \beta$. FIND LARGEST $\beta$ THAT MAKES HAVING A PIECE AN $s_V$-piece THAT IS $> \beta$ IMPLIES SOME PIECE IS $\le \frac{33}{70}$.

**Case 3.3** Both of the following hold:

- All of the $s_V$-pieces are in the interval $(\frac{33}{70}, \alpha]$.
- All of the $s_{V-1}$-pieces are in the interval $[\beta, \frac{37}{70}]$.

Hence there are no pieces in $(\alpha, \beta)$. By looking at the buddies of these pieces there are no pieces in $(1-\beta, 1-\alpha)$.

FROM THIS POINT ON, YOU ARE ON YOUR OWN.

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4. (30 points) Reread the proof that, for all $c$, there exists a graph $G_c$ such that $\chi(G_c) = c$ and $g(G_c) = 6$. Let $M_c$ be the number of vertices in $G_c$.

   (a) Write a recurrence for the $M_c$ in terms of $M_{c-1}$.

   (b) Give an asymptotic expression for $M_c$.

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5. (0 points– For your own Enlightenment) Reread the proof that, for all $c$, there exists a graph $G_c$ such that $\chi(G_c) = c$ and $g(G_c) = 9$. It used PVDW and *easy number theory.*

   Write down a theorem in Number Theory whose truth would show that the construction yields $g(G_c) \geq 12$.

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6. (10 points) For each of the following say if its an APPLICATION!, an "application" or a "are you kidding me?" Explain why you think so.

   (a) Using Ramsey Theory to prove the Bolzano-Weierstrass Theorem.

   (b) Using Hypergraph Ramsey to prove Can Ramsey.

   (c) Using Ramsey to assist program checkers to prove programs halt.

   (d) Using Can Ramsey to show that for all infinite sets of points in the plane there is an infinite subset where all distances are distinct.

   (e) Using Ramsey to show that if $(X, \preceq)$ is a wqo then every sequence has an infinite increasing subsequence.

   (f) Using Ramsey to show that, for all $k$, there exists $N$, so given any $N$ points in the plane, no 3 colinear, there is a set of $k$ that form a convex $k$-gon.

   (g) Using Ramsey to show that in the language of graphs (or colored $\le a$-ary hypergraphs) the spectrum problem for $\exists^*\forall^*$ sentences is decidable.

   (h) Using Ramsey to show that if the universe is big enough then tables should be sorted.

   (i) Using Poly VDW to construct graphs $G$ with $\chi(G) = c$ and $g(G) = 9$.

   (j) Using VDW to show there that, for all $p$, there exists long sequences of consecutive squares mod $p$ (if I get to it).