Please Fill Out All of Your Courses Teaching Evals

May 10, 2022
Please Fill Out All of Your Courses Teaching Evals

1) Teachers read their teaching eval comments for self-improvement.
2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.
3) The Undergrad Chair reads others teaching evals and may discuss it with the teacher.
4) The committee that gives out teaching awards reads the teaching evals to help make a decision.
5) When a teacher goes up for tenure, the teaching evals are used in the teaching report.
6) The biggest problem we have for all of the above is when not that many students fill them out. Hence
7) Please Fill Out the Teaching Evals in All of your Courses
1) Teachers read their teaching eval comments for self-improvement.
Please Fill Out All of Your Courses Teaching Evals

1) Teachers read their teaching eval comments for self-improvement.
2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.
Please Fill Out All of Your Courses Teaching Evals

1) Teachers read their teaching eval comments for self-improvement.
2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.
3) The Undergrad Chair reads others teaching evals and may discuss it with the teacher.
Please Fill Out All of Your Courses Teaching Evals

1) Teachers read their teaching eval comments for self-improvement.
2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.
3) The Undergrad Chair reads others teaching evals and may discuss it with the teacher.
4) The committee that gives out teaching awards reads the teaching evals to help make a decision.

5) When a teacher goes up for tenure, the teaching evals are used in the teaching report.
6) The biggest problem we have for all of the above is when not that many students fill them out. Hence...
Please Fill Out All of Your Courses Teaching Evals

1) Teachers read their teaching eval comments for self-improvement.
2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.
3) The Undergrad Chair reads others teaching evals and may discuss it with the teacher.
4) The committee that gives out teaching awards reads the teaching evals to help make a decision.
5) When a teacher goes up for tenure, the teaching evals are used in the teaching report.
1) Teachers read their teaching eval comments for self-improvement.
2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.
3) The Undergrad Chair reads others teaching evals and may discuss it with the teacher.
4) The committee that gives out teaching awards reads the teaching evals to help make a decision.
5) When a teacher goes up for tenure, the teaching evals are used in the teaching report.
6) The biggest problem we have for all of the above is when not that many students fill them out. Hence
1) Teachers read their teaching eval comments for self-improvement.
2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.
3) The Undergrad Chair reads others teaching evals and may discuss it with the teacher.
4) The committee that gives out teaching awards reads the teaching evals to help make a decision.
5) When a teacher goes up for tenure, the teaching evals are used in the teaching report.
6) The biggest problem we have for all of the above is when not that many students fill them out. Hence
7) **Please Fill Out the Teaching Evals in All of your Courses**
Topics Not Covered in Grad Ramsey 2022

Exposition by William Gasarch

May 10, 2022
We Didn’t Cover X Because . . .

What topics in Ramsey theory didn’t we cover?
We Didn’t Cover X Because... 

What topics in Ramsey theory didn’t we cover?

Why didn’t we cover them?
We Didn’t Cover X Because... 

What topics in Ramsey theory didn’t we cover?

Why didn’t we cover them?

- Could cover if I had more time. Might cover it next time.
We Didn’t Cover X Because. . .

What topics in Ramsey theory didn’t we cover?
Why didn’t we cover them?

- Could cover if I had more time. Might cover it next time.
- Bill just not that interested.
We Didn’t Cover X Because... 

What topics in Ramsey theory didn’t we cover?

Why didn’t we cover them?

- Could cover if I had more time. Might cover it next time.
- Bill just not that interested.
- Too hard for Students.
What topics in Ramsey theory didn’t we cover?

Why didn’t we cover them?

- Could cover if I had more time. Might cover it next time.
- Bill just not that interested.
- Too hard for Students.
- Too hard for Bill.
We Didn’t Cover X Because. . .

What topics in Ramsey theory didn’t we cover?

Why didn’t we cover them?

▷ Could cover if I had more time. Might cover it next time.
▷ Bill just not that interested.
▷ Too hard for Students.
▷ Too hard for Bill.
▷ Some combination of the above.
Could Have Covered: VDW

Exposition by William Gasarch

May 10, 2022
Canonical VDW

**Can VDW** For all $k$ there exists $W = W(k)$ such that for any COL: $[W] \rightarrow [\omega]$ there exists $a, d$ such that either

\[ a, a + d, \ldots, a + (k - 1)d \text{ are all the same color} \]

or

\[ a, a + d, \ldots, a + (k - 1)d \text{ are all different colors} \]
Canonical VDW

**Can VDW** For all $k$ there exists $W = W(k)$ such that for any \( \text{COL}: [W] \to [\omega] \) there exists $a, d$ such that either

\[
a, a + d, \ldots, a + (k - 1)d \text{ are all the same color}
\]

or

\[
a, a + d, \ldots, a + (k - 1)d \text{ are all different colors}
\]

Nice proof, but it relies on 2-dim VDW thm.
Canonical VDW

**Can VDW** For all $k$ there exists $W = W(k)$ such that for any
\[ \text{COL}: [W] \rightarrow [\omega] \]
there exists $a, d$ such that either

\[
a, a + d, \ldots, a + (k - 1)d \text{ are all the same color}
\]

or

\[
a, a + d, \ldots, a + (k - 1)d \text{ are all different colors}
\]

▶ Nice proof, but it relies on 2-dim VDW thm.
▶ There is a version for multidim VDW, but its messy.
**Canonical VDW**

**Can VDW** For all $k$ there exists $W = W(k)$ such that for any $\text{COL} : [W] \to [\omega]$ there exists $a, d$ such that either

$$a, a + d, \ldots, a + (k - 1)d$$

are all the same color

or

$$a, a + d, \ldots, a + (k - 1)d$$

are all different colors

- Nice proof, but it relies on 2-dim VDW thm.
- There is a version for multidim VDW, but it's messy.
- Certainly could have taught this this semester.
Canonical VDW

**Can VDW** For all $k$ there exists $W = W(k)$ such that for any $\text{COL} : [W] \rightarrow [\omega]$ there exists $a, d$ such that either

$a, a + d, \ldots, a + (k - 1)d$ are all the same color

or

$a, a + d, \ldots, a + (k - 1)d$ are all different colors

- Nice proof, but it relies on 2-dim VDW thm.
- There is a version for multidim VDW, but its messy.
- Certainly could have taught this semester.

**Research** Better bounds on Can VDW Numbers.
Interesting App of VDWs Thm to Number Theory

Can use Extended VDW’s thm to prove the following

- For all $k$ there exists $p_0$ such that for all primes $p \geq p_0$ there are $k$ consecutive squares mod $p$.
- For all $k$ there exists $p_0$ such that for all primes $p \geq p_0$ there are $k$ consecutive non-squares mod $p$.


I could have proven this in class and might next time I teach it. Research The proof gives VDW-like bounds. Hard NT gives better bounds. Get better bounds in elementary way.
Interesting App of VDWs Thm to Number Theory

Can use Extended VDW’s thm to prove the following

- For all $k$ there exists $p_o$ such that for all primes $p \geq p_o$ there are $k$ consecutive squares mod $p$. 
Interesting App of VDWs Thm to Number Theory

Can use Extended VDW’s thm to prove the following

- For all $k$ there exists $p_o$ such that for all primes $p \geq p_o$ there are $k$ consecutive squares mod $p$.
- For all $k$ there exists $p_o$ such that for all primes $p \geq p_o$ there are $k$ consecutive non-squares mod $p$.

I could have proven this in class and might next time I teach it.

Research

Interesting App of VDWs Thm to Number Theory

Can use Extended VDW’s thm to prove the following

- For all $k$ there exists $p_o$ such that for all primes $p \geq p_o$ there are $k$ consecutive squares mod $p$.
- For all $k$ there exists $p_o$ such that for all primes $p \geq p_o$ there are $k$ consecutive non-squares mod $p$.

- [Link to the blog post](https://blog.computationalcomplexity.org/2009/08/application-of-vdw-theorem-to-number.html)

I could have proven this in class and might next time I teach it.
Interesting App of VDWs Thm to Number Theory

Can use Extended VDW’s thm to prove the following

▶ For all $k$ there exists $p_o$ such that for all primes $p \geq p_o$ there are $k$ consecutive squares mod $p$.

▶ For all $k$ there exists $p_o$ such that for all primes $p \geq p_o$ there are $k$ consecutive non-squares mod $p$.


I could have proven this in class and might next time I teach it.

Alice, Bob, Carol have an $n$-bit number on their foreheads: $A, B, C$. They want to know if $A + B + C = 2^n - 1$. They can do this by communicating $n$ bits. Can they do better? Using large sets without 3-APs can show they can do this in $\sim \sqrt{n}$ bits. Lower bound of $\Omega(\log \log n)$ (By Gasarch! Honest!) Could have done this and have in past semesters.
Alice, Bob, Carol have an $n$-bit number on their foreheads: $A, B, C$. They want to know if $A + B + C = 2^n - 1$. Using large sets without 3-APs can show they can do this in $\sim \sqrt{n}$ bits. Lower bound of $\Omega(\log \log n)$ (By Gasarch! Honest!) Certainly could have done this and have in past semesters.
Alice, Bob, Carol have an $n$-bit number on their foreheads: $A, B, C$. They want to know if $A + B + C = 2^n - 1$. They can do this by communicating $n$ bits. Can they do better?
Alice, Bob, Carol have an $n$-bit number on their foreheads: $A, B, C$. They want to know if $A + B + C = 2^n - 1$. They can do this by communicating $n$ bits. Can they do better? Using large sets without 3-APs can show they can do this in $\sim \sqrt{n}$ bits.
Interesting App of VDWs Thm Comm. Comp

Alice, Bob, Carol have an $n$-bit number on their foreheads: $A, B, C$. They want to know if $A + B + C = 2^n - 1$.

They can do this by communicating $n$ bits. Can they do better?

Using large sets without 3-APs can show they can do this in $\sim \sqrt{n}$ bits.

Lower bound of $\Omega(\log \log n)$ (By Gasarch! Honest!)
Alice, Bob, Carol have an $n$-bit number on their foreheads: $A, B, C$. They want to know if $A + B + C = 2^n - 1$.
They can do this by communicating $n$ bits. Can they do better?
Using large sets without 3-APs can show they can do this in $\sim \sqrt{n}$ bits.
Lower bound of $\Omega(\log \log n)$ (By Gasarch! Honest!)
Certainly could have done this and have in past semesters.
Folkman’s Thm

**Rado’s Thm** Let \( a_1, \ldots, a_k \in \mathbb{Z} \). TFAE

- Some subset of the \( a_i \)’s sums to 0.
- For all \( c \), for all \( \text{COL} : \mathbb{N} \to [c] \) there exists mono solution to

\[
a_1x_1 + \cdots + a_kx_k = 0.
\]
Folkman’s Thm

Rado’s Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the $a_i$’s sums to 0.
- For all $c$, for all $\text{COL}: \mathbb{N} \to [c]$ there exists mono solution to

$$a_1x_1 + \cdots + a_kx_k = 0.$$ 

There is a version for a particular systems of linear equations.
Folkman’s Thm

Rado’s Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the $a_i$’s sums to 0.
- For all $c$, for all $\text{COL}: \mathbb{N} \to [c]$ there exists mono solution to
  
  $$a_1x_1 + \cdots + a_kx_k = 0.$$ 

There is a version for a particular systems of linear equations.

Folkman’s Thm For all $k, c$ there exists $N = N(k, c)$ such that for all $\text{COL}: [N] \to [c]$ there exists $x_1, \ldots, x_k$ such that ALL non-empty sums of the $x_i$’s are the same color.
Rado’s Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

1. Some subset of the $a_i$’s sums to 0.
2. For all $c$, for all $\text{COL}: \mathbb{N} \to [c]$ there exists a mono solution to

$$a_1x_1 + \cdots + a_kx_k = 0.$$ 

There is a version for a particular systems of linear equations.

Folkman’s Thm For all $k, c$ there exists $N = N(k, c)$ such that for all $\text{COL}: [N] \to [c]$ there exists $x_1, \ldots, x_k$ such that ALL non-empty sums of the $x_i$’s are the same color.

Great thm, nice proof. Might cover it in the future.
Research Questions

▶ Better Bounds on Rado and Folkman Numbers.
▶ Given a coloring actually FIND mono solution.
▶ Example

$$4x + 5y = 10z$$

$$\exists \text{COL}: \mathbb{N} \rightarrow \mathbb{6} \text{ with no mono sol.}$$

$$\forall \text{COL}: \mathbb{N} \rightarrow \mathbb{1} \text{ is mono sol.}$$

Find $$c$$ such that

$$\exists \text{COL}: \mathbb{N} \rightarrow \mathbb{c} \text{ with no mono sol.}$$

$$\forall \text{COL}: \mathbb{N} \rightarrow \mathbb{c-1} \text{ is mono sol.}$$

Can ask this question for many equations.

▶ Canonical Version of Rado or Folkman’s Thm.
▶ Caution: Some of this may be known.
Research Questions

▶ Better Bounds on Rado and Folkman Numbers.
Research Questions

- Better Bounds on Rado and Folkman Numbers.
- Given a coloring actually FIND mono solution.
Research Questions

- Better Bounds on Rado and Folkman Numbers.
- Given a coloring actually FIND mono solution.
- Example $4x + 5y = 10z$:

  $\exists \text{COL}: \mathbb{N} \rightarrow [6]$ with no mono sol.

  $\forall \text{COL}: \mathbb{N} \rightarrow [1]$ is mono sol.

  Find $c$ such that $\exists \text{COL}: \mathbb{N} \rightarrow [c]$ with no mono sol.

  $\forall \text{COL}: \mathbb{N} \rightarrow [c-1]$ is mono sol.

  Can ask this question for many equations.

- Canonical Version of Rado or Folkman’s Thm.
- Caution: Some of this may be known.
Research Questions

- Better Bounds on Rado and Folkman Numbers.
- Given a coloring actually FIND mono solution.
- **Example** $4x + 5y = 10z$:
  - $\exists \text{COL}: [\mathbb{N}] \to [6]$ with no mono sol.
  - $\forall \text{COL}: [\mathbb{N}] \to [1]$ is mono sol.
Better Bounds on Rado and Folkman Numbers.

Given a coloring actually FIND mono solution.

Example $4x + 5y = 10z$:

$\exists$ COL: $[\mathbb{N}] \to [6]$ with no mono sol.
$\forall$ COL: $[\mathbb{N}] \to [1]$ is mono sol.

Find $c$ such that

$\exists$ COL: $[\mathbb{N}] \to [c]$ with no mono sol.
$\forall$ COL: $[\mathbb{N}] \to [c - 1]$ is mono sol.

Can ask this question for many equations.
Research Questions

▶ Better Bounds on Rado and Folkman Numbers.
▶ Given a coloring actually FIND mono solution.
▶ **Example** $4x + 5y = 10z$:
  $\exists \text{COL}: [\mathbb{N}] \rightarrow [6]$ with no mono sol.
  $\forall \text{COL}: [\mathbb{N}] \rightarrow [1]$ is mono sol.

Find $c$ such that
$\exists \text{COL}: [\mathbb{N}] \rightarrow [c]$ with no mono sol.
$\forall \text{COL}: [\mathbb{N}] \rightarrow [c - 1]$ is mono sol.
Can ask this question for many equations.
▶ Canonical Version of Rado or Folkman’s Thm.
Research Questions

▶ Better Bounds on Rado and Folkman Numbers.
▶ Given a coloring actually FIND mono solution.
▶ **Example** $4x + 5y = 10z$:
  $\exists$ COL: $[N] \rightarrow [6]$ with no mono sol.
  $\forall$ COL: $[N] \rightarrow [1]$ is mono sol.

Find $c$ such that
  $\exists$ COL: $[N] \rightarrow [c]$ with no mono sol.
  $\forall$ COL: $[N] \rightarrow [c - 1]$ is mono sol.

Can ask this question for many equations.

▶ Canonical Version of Rado or Folkman’s Thm.
▶ Caution: Some of this may be known.
Hilbert’s Cube Lemma For all $k, c$ there exists $H = H(k, c)$ such that for all $\text{COL}: [H] \to [c]$ there exists $x_0, x_1, \ldots, x_k$ such that

$$\left\{ x_0 + \sum_{i=1}^{k} b_i x_i : b_i \in \{0, 1\} \right\}$$

is monochromatic.
Hilbert’s Cube Lemma

For all \( k, c \) there exists \( H = H(k, c) \) such that for all COL: \([H] \rightarrow [c] \) there exists \( x_0, x_1, \ldots, x_k \) such that

\[
\{ x_0 + \sum_{i=1}^{k} b_i x_i : b_i \in \{0, 1\} \}
\]

is monochromatic. Hilbert saw it as a Lemma to prove:
Hilbert’s Cube Lemma

For all $k, c$ there exists $H = H(k, c)$ such that for all $\text{COL}: [H] \rightarrow [c]$ there exists $x_0, x_1, \ldots, x_k$ such that

$$\{x_0 + \sum_{i=1}^{k} b_i x_i : b_i \in \{0, 1\}\}$$

is monochromatic. Hilbert saw it as a Lemma to prove:

H Irreducibility Thm (2 var case). If $p(x, y) \in \mathbb{Q}[x, y]$ is irred then there exists $a \in \mathbb{Z}$ such that $p(x, a) \in \mathbb{Q}[x]$ is irred.
Hilbert’s Cube Lemma

For all \(k, c\) there exists \(H = H(k, c)\) such that for all \(\text{COL}: [H] \to [c]\) there exists \(x_0, x_1, \ldots, x_k\) such that

\[
\{ x_0 + \sum_{i=1}^{k} b_i x_i : b_i \in \{0, 1\} \}
\]

is monochromatic. Hilbert saw it as a Lemma to prove:

**H Irreducibility Thm** (2 var case). If \(p(x, y) \in \mathbb{Q}[x, y]\) is irred then there exists \(a \in \mathbb{Z}\) such that \(p(x, a) \in \mathbb{Q}[x]\) is irred.

▶ HIT is an important thm and a legit app (to math).
Hilbert’s Cube Lemma

For all $k, c$ there exists $H = H(k, c)$ such that for all $\text{COL}: [H] \to [c]$ there exists $x_0, x_1, \ldots, x_k$ such that

$$\{x_0 + \sum_{i=1}^{k} b_i x_i : b_i \in \{0, 1\}\}$$

is monochromatic. Hilbert saw it as a Lemma to prove:

**H Induction (2 var case).** If $p(x, y) \in \mathbb{Q}[x, y]$ is irreducible then there exists $a \in \mathbb{Z}$ such that $p(x, a) \in \mathbb{Q}[x]$ is irreducible.

- HIT is an important theorem and a legit app (to math).
- As the FIRST Ramseyian Thm it is important historically.
Hilbert’s Cube Lemma

For all \( k, c \) there exists \( H = H(k, c) \) such that for all \( \text{COL} : [H] \to [c] \) there exists \( x_0, x_1, \ldots, x_k \) such that

\[
\{ x_0 + \sum_{i=1}^{k} b_i x_i : b_i \in \{0, 1\} \}
\]

is monochromatic. Hilbert saw it as a Lemma to prove:

**H Hilbert’s Cube Lemma (2 var case).** If \( p(x, y) \in \mathbb{Q}[x, y] \) is irred then there exists \( a \in \mathbb{Z} \) such that \( p(x, a) \in \mathbb{Q}[x] \) is irred.

- HIT is an important thm and a legit app (to math).
- As the FIRST Ramseyian Thm it is important historically.
- Only presentation in English with modern notation Villarino, Gasarch, Regan: \url{https://arxiv.org/abs/1611.06303}
Hilbert’s Cube Lemma

For all \( k, c \) there exists \( H = H(k, c) \) such that for all \( \text{COL} : [H] \to [c] \) there exists \( x_0, x_1, \ldots, x_k \) such that

\[
\{ x_0 + \sum_{i=1}^{k} b_i x_i : b_i \in \{0, 1\} \}
\]

is monochromatic. Hilbert saw it as a Lemma to prove:

H Irreducibility Thm (2 var case). If \( p(x, y) \in \mathbb{Q}[x, y] \) is irred then there exists \( a \in \mathbb{Z} \) such that \( p(x, a) \in \mathbb{Q}[x] \) is irred.

- HIT is an important thm and a legit app (to math).
- As the FIRST Ramseyian Thm it is important historically.
- Only presentation in English with modern notation Villarino, Gasarch, Regan: https://arxiv.org/abs/1611.06303
- I’ve taught before and could teach again.
Roth’s Thm

Roth’s Theorem was prove in 1954 and is a special case of Szemeredi’s Theorem which was proven in 1974.

Roth’s Thm Every set of upper positive density has a 3-AP.
Roth’s Thm

Roth’s Theorem was prove in 1954 and is a special case of Szemeredi’s Theorem which was proven in 1974.

**Roth’s Thm** Every set of upper positive density has a 3-AP.

- There is a combinatorial proof that I have taught in the past.
Roth’s Thm

Roth’s Theorem was proved in 1954 and is a special case of Szemeredi’s Theorem which was proven in 1974.

**Roth’s Thm** Every set of upper positive density has a 3-AP.

- There is a combinatorial proof that I have taught in the past.
- There is an analytic proof which is the easiest analytic proof of results of this type, so perhaps could and should be taught.
Roth’s Thm

Roth’s Theorem was proved in 1954 and is a special case of Szemeredi’s Theorem which was proven in 1974.

Roth’s Thm Every set of upper positive density has a 3-AP.

- There is a combinatorial proof that I have taught in the past.

- There is an analytic proof which is the easiest analytic proof of results of this type, so perhaps could and should be taught.

- There is a computer-assisted proof
  This is Roth’s proof done with the ideas showing and the computation rightly put into the background.
Roth’s Thm

Roth’s Theorem was proved in 1954 and is a special case of Szemeredi’s Theorem which was proven in 1974.

**Roth’s Thm** Every set of upper positive density has a 3-AP.

▶ There is a combinatorial proof that I have taught in the past.

▶ There is an analytic proof which is the easiest analytic proof of results of this type, so perhaps could and should be taught.

▶ There is a computer-assisted proof


This is Roth’s proof done with the ideas showing and the computation rightly put into the background.

▶ **Research** Get better bounds: How big a subset of \{1,\ldots,1000\} before guaranteed a 3-AP? 4-AP? etc.
A Stupid App of Schur’s Thm to Number Theory

Schur’s Theorem is a special case of Rado’s Theorem.

**Schur’s Thm** For all $c$ there exists $S = S(c)$ such that for all $\text{COL} : [S] \to [c]$ there exists $x, y, z$ same color such that $x + y = z$. 

FLT For all $n \geq 3$ there do not exists $x, y, z \in \mathbb{N}$ such that $x^n + y^n = z^n$. (The $n = 4$ case was done by Fermat.)

Gasarch proved: Thm *(Schur’s Thm + FLT(4) implies there are an infinite number of primes.*

Schur’s Theorem is a special case of Rado’s Theorem.

**Schur’s Thm** For all $c$ there exists $S = S(c)$ such that for all $\text{COL}: [S] \rightarrow [c]$ there exists $x, y, z$ same color such that $x + y = z$.

**FLT** For all $n \geq 3$ there does not exist $x, y, z \in \mathbb{N}$ such that $x^n + y^n = z^n$. (The $n = 4$ case was done by Fermat.)
Schur’s Theorem is a special case of Rado’s Theorem.

**Schur’s Thm** For all \( c \) there exists \( S = S(c) \) such that for all \( \text{COL}: [S] \to [c] \) there exists \( x, y, z \) same color such that \( x + y = z \).

**FLT** For all \( n \geq 3 \) there does not exist \( x, y, z \in \mathbb{N} \) such that \( x^n + y^n = z^n \). (The \( n = 4 \) case was done by Fermat.)

Gasarch proved:

**Thm** (Schur’s Thm + FLT(4) implies there are an infinite number of primes. [https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/schurfilt.pdf](https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/schurfilt.pdf)
Rado’s Theorem over the Reals

Vote
For all \( COL : \mathbb{R} \to \mathbb{N} \) there exists \( w, x, y, z \) all the same color:

\[
w + x = y + z
\]

- TRUE
- FALSE
- OTHER

OTHER: Statement is equiv to \( \neg CH \) and hence is Ind of ZFC. Proven by Erdos. Write up by Fenner and Gasarch is here: [http://www.cs.umd.edu/~gasarch/BLOGPAPERS/radozfc.pdf](http://www.cs.umd.edu/~gasarch/BLOGPAPERS/radozfc.pdf)
Rado’s Theorem over the Reals

Vote
For all \( COL : \mathbb{R} \rightarrow \mathbb{N} \) there exists \( w, x, y, z \) all the same color:

\[ w + x = y + z \]

- TRUE
- FALSE
- OTHER

OTHER: Statement is equiv to \( \neg CH \) and hence is Ind of ZFC.
Rado’s Theorem over the Reals

Vote
For all \( COL: \mathbb{R} \to \mathbb{N} \) there exists \( w, x, y, z \) all the same color:

\[ w + x = y + z \]

- TRUE
- FALSE
- OTHER

OTHER: Statement is equiv to \( \neg CH \) and hence is Ind of ZFC.

Proven by Erdos. Write up by Fenner and Gasarch is here:
Could have Covered: Ramsey

Exposition by William Gasarch

May 10, 2022
Other Ramsey Numbers

$R(C_k)$ is least $n$ such that for all 2-coloring of $[n]^2$ there exists monochromatic $k$-cycle.

Sample Thm

\[
R(C_k) = \begin{cases} 
6 & \text{if } k = 3 \text{ or } k = 4 \\
2k - 1 & \text{if } k \geq 5 \text{ and } k \equiv 1 \pmod{2} \\
3k^2 - 1 & \text{if } k \geq 4 \text{ and } k \equiv 0 \pmod{2}
\end{cases}
\] (1)

There are many results and the proofs are elementary. I would need to learn it (this is a PRO). I may have a student writeup the proofs for a project. Then I'll see if it's interesting.

For every result of this type see https://www.combinatorics.org/files/Surveys/ds1/ds1v15-2017.pdf
Other Ramsey Numbers

\( R(C_k) \) is least \( n \) such that for all 2-coloring of \( \binom{[n]}{2} \) there exists monochromatic \( k \)-cycle.
Other Ramsey Numbers

\( R(C_k) \) is least \( n \) such that for all 2-coloring of \( \binom{n}{2} \) there exists monochromatic \( k \)-cycle.

**Sample Thm**

\[
R(C_k) = \begin{cases} 
6 & \text{if } k = 3 \text{ or } k = 4 \\
2k - 1 & \text{if } k \geq 5 \text{ and } k \equiv 1 \pmod{2} \\
\frac{3k}{2} - 1 & \text{if } k \geq 4 \text{ and } k \equiv 0 \pmod{2}
\end{cases}
\]  (1)

Their are many results and the proofs are elementary. I would need to learn it (this is a PRO). I may have a student writeup the proofs for a project. Then I'll see if its interesting.

Other Ramsey Numbers

\( R(C_k) \) is least \( n \) such that for all 2-coloring of \( \binom{[n]}{2} \) there exists monochromatic \( k \)-cycle.

**Sample Thm**

\[
R(C_k) = \begin{cases} 
6 & \text{if } k = 3 \text{ or } k = 4 \\
2k - 1 & \text{if } k \geq 5 \text{ and } k \equiv 1 \pmod{2} \\
\frac{3k}{2} - 1 & \text{if } k \geq 4 \text{ and } k \equiv 0 \pmod{2}
\end{cases}
\]  

(1)

▶ There are many results and the proofs are elementary.
Other Ramsey Numbers

\( R(C_k) \) is least \( n \) such that for all 2-coloring of \( \binom{n}{2} \) there exists monochromatic \( k \)-cycle.

**Sample Thm**

\[
R(C_k) = \begin{cases} 
6 & \text{if } k = 3 \text{ or } k = 4 \\
2k - 1 & \text{if } k \geq 5 \text{ and } k \equiv 1 \pmod{2} \\
\frac{3k}{2} - 1 & \text{if } k \geq 4 \text{ and } k \equiv 0 \pmod{2}
\end{cases}
\]  
(1)

- There are many results and the proofs are elementary.
- I would need to learn it (this is a PRO). I may have a student writeup the proofs for a project. Then I’ll see if its interesting.
Other Ramsey Numbers

$R(C_k)$ is least $n$ such that for all 2-coloring of $\left(\binom{n}{2}\right)$ there exists monochromatic $k$-cycle.

Sample Thm

$$R(C_k) = \begin{cases} 6 & \text{if } k = 3 \text{ or } k = 4 \\ 2k - 1 & \text{if } k \geq 5 \text{ and } k \equiv 1 \pmod{2} \\ \frac{3k}{2} - 1 & \text{if } k \geq 4 \text{ and } k \equiv 0 \pmod{2} \end{cases} \quad (1)$$

- There are many results and the proofs are elementary.
- I would need to learn it (this is a PRO). I may have a student writeup the proofs for a project. Then I’ll see if it’s interesting.
- For every result of this type see https://www.combinatorics.org/files/Surveys/ds1/ds1v15-2017.pdf
Research Projects

- Actually FIND the colorings.
- Simplify or unify the proofs
- **Ramsey Games** Example: Parameter $k, n$. Players RED and BLUE alternate coloring the edges of $K_n$. RED goes first. The first player to get a $C_k$ in their color wins.
  1. For which $n$ does RED have a winning strategy?
  2. Design an ML to play this well (my REU project)
  3. EVERY thm in Ramsey Thm (and the VDW part) can be made into a game and lead to research projects.
Better Bounds on 3-Hypergraph Ramsey

We did:

\[ R_3(k) \leq 2^{2^k} \]

Better is known:

\[ R_3(k) \leq 2^{2^{2^k}} \]

The proof is nice but long. I would need to refresh on it to see if appropriate.

Research

Use their technique on other Ramsey problems.
We did:

\textbf{Thm} \hspace{1mm} R_3(k) \leq 2^{2^4k}.
Better Bounds on 3-Hypergraph Ramsey

We did:

**Thm** $R_3(k) \leq 2^{2^4k}$.

Better is known:

**Thm** $R_3(k) \leq 2^{2^{2k}}$. 
We did:

**Thm** $R_3(k) \leq 2^{2^4 k}$.

Better is known:

**Thm** $R_3(k) \leq 2^{2^{2^k}}$.

The proof is nice but long.
Better Bounds on 3-Hypergraph Ramsey

We did:

**Thm** $R_3(k) \leq 2^{24k}$.

Better is known:

**Thm** $R_3(k) \leq 2^{2^{2k}}$.

The proof is nice but long.

I would need to refresh on it to see if appropriate.
We did:

**Thm** $R_3(k) \leq 2^{2^k}$.

Better is known:

**Thm** $R_3(k) \leq 2^{2^{2k}}$.

The proof is nice but long.

I would need to refresh on it to see if appropriate.

**Research** Use their technique on other Ramsey problems.
Lefmann and Rodl proved
\[ \text{Thm } CR(k) \leq 2^{O(k^2 \log k)}. \]
Better Bounds on Can Ramsey

Lefmann and Rodl proved

\textbf{Thm} \( CR(k) \leq 2^{O(k^2 \log k)} \).


▶ Proof is Mileti-style.

▶ I have written up the construction. It's in my treatise on Can Ramsey: https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/canramsey.pdf
Better Bounds on Can Ramsey

Lefmann and Rodl proved

**Thm**  $CR(k) \leq 2^{O(k^2 \log k)}$.

- Proof is Mileti-style.
Lefmann and Rodl proved

**Thm** $CR(k) \leq 2^{O(k^2 \log k)}$.

- Proof is Mileti-style.
- I have written up the construction. It’s in my treatise on Can Ramsey: [https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/canramsey.pdf](https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/canramsey.pdf)
Lefmann and Rodl proved

**Thm** \( CR(k) \leq 2^{O(k^2 \log k)} \).

- Proof is Mileti-style.
- I have written up the construction. It’s in my treatise on Can Ramsey: [https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/canramsey.pdf](https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes/canramsey.pdf)
- Do we really need more Can Ramsey in the course?
The following is well known; however, I may be the first person to write down the proof.


**Thm** For all $k$ there exists $n = n(k)$ such that for all COL: $(\binom{k,\ldots,n}{2}) \rightarrow [\omega]$ there is a large set that is either homog, min-homog, max-homog, rainbow.
The following is well known; however, I may be the first person to write down the proof.

Thm For all $k$ there exists $n = n(k)$ such that for all $\text{COL} : \binom{\{k, \ldots, n\}}{2} \rightarrow [\omega]$ there is a large set that is either homog, min-homog, max-homog, rainbow.

- Main Thm has inf-can, fin-can, large-can as corollaries.
Large Can Ramsey

The following is well known; however, I may be the first person to write down the proof.

**Thm** For all $k$ there exists $n = n(k)$ such that for all $\text{COL}: \binom{\{k, \ldots, n\}}{2} \to [\omega]$ there is a large set that is either homog, min-homog, max-homog, rainbow.

- Main Thm has inf-can, fin-can, large-can as corollaries.
- Thm is mostly the proof of can from 4-hypergraph Ramsey.
The following is well known; however, I may be the first person to write down the proof.


**Thm** For all $k$ there exists $n = n(k)$ such that for all $\text{COL}: (\{k, \ldots, n\}^2) \rightarrow [\omega]$ there is a large set that is either homog, min-homog, max-homog, rainbow.

- Main Thm has inf-can, fin-can, large-can as corollaries.
- Thm is mostly the proof of can from 4-hypergraph Ramsey.
- Bounds on $n(k)$ are in terms of the $LR_4$. 
The following is well known; however, I may be the first person to write down the proof.


**Thm** For all $k$ there exists $n = n(k)$ such that for all $\text{COL}: \binom{\{k, \ldots, n\}}{2} \rightarrow [\omega]$ there is a large set that is either homog, min-homog, max-homog, rainbow.

▶ Main Thm has inf-can, fin-can, large-can as corollaries.

▶ Thm is mostly the proof of can from 4-hypergraph Ramsey.

▶ Bounds on $n(k)$ are in terms of the $LR_4$.

**Research** Get the bound in terms of $LR_3$ or lower.
For all $a, k \in \mathbb{N}$ there exist $C = C(a, k)$ such that for all $\text{COL} : [\binom{[a]}{C}] \rightarrow [\omega]$ there exists a set $H, |H| = k$ and $1 \leq i_1 < \cdots < i_L \leq a$ such that for all $p_1 < \cdots < p_a \in H$ and $q_1 < \cdots < q_a \in H$

$\text{COL}(p_1, \ldots, p_a) = \text{COL}(q_1, \ldots, q_a)$ iff $(p_{i_1}, \ldots, p_{i_L}) = (q_{i_1}, \ldots, q_{i_L})$

- Similar to the proof on graphs, but messier.
- *On canonical Ramsey numbers for coloring three-element sets* by Lefmann and Rodl behind paywalls, lost to humanity.
- Optimal results due to Shelah:

**Research** Give easier proofs of bounds.
Could have Covered: Euclidean Ramsey Theory

Exposition by William Gasarch

May 10, 2022
Euclidean Ramsey Theory

**Sample Thm** Let $T$ be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of $\mathbb{R}^2$ there exists three points that form triangle $T$ (note- actually form $T$, not just similar to $T$) that are monochromatic.
Euclidean Ramsey Theory

Sample Thm Let \( T \) be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of \( \mathbb{R}^2 \) there exists three points that form triangle \( T \) (note- actually form \( T \), not just similar to \( T \)) that are monochromatic.

▶ Just getting to interesting results would a take while.
Euclidean Ramsey Theory

**Sample Thm** Let $T$ be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of $\mathbb{R}^2$ there exists three points that form triangle $T$ (note- actually form $T$, not just similar to $T$) that are monochromatic.

- Just getting to interesting results would a take while.
- Would need to dump some other topic.

Euclidean Ramsey Theory

Sample Thm Let $T$ be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of $\mathbb{R}^2$ there exists three points that form triangle $T$ (note- actually form $T$, not just similar to $T$) that are monochromatic.

- Just getting to interesting results would a take while.
- Would need to dump some other topic.
- Bill would need to relearn the material (this is a PRO).
Euclidean Ramsey Theory

Sample Thm Let $T$ be a triangle with a 30, 90, or 150 degree angle. For every 2-coloring of $\mathbb{R}^2$ there exists three points that form triangle $T$ (note- actually form $T$, not just similar to $T$) that are monochromatic.

- Just getting to interesting results would a take while.
- Would need to dump some other topic.
- Bill would need to relearn the material (this is a PRO).
- For more:
Results Bill Likes But Would be Hard to Teach: VDW

Exposition by William Gasarch

May 10, 2022
Def \( L \) is a language. Game:

\begin{itemize}
  \item Alice is Poly time and she has \( x \), \(|x| = n\).
  \item Bob is all powerful and he has nothing.
  \item They cooperate to determine if \( x \in L \).
    Alice could just send Bob \( x \). That takes \( n \) bits.
\end{itemize}
Def $L$ is a language. Game:

- Alice is Poly time and she has $x$, $|x| = n$.
App of 3-Free Sets to Complexity Theory

**Def** $L$ is a language. Game:
- Alice is Poly time and she has $x$, $|x| = n$.
- Bob is all powerful and he has nothing.
App of 3-Free Sets to Complexity Theory

**Def** \( L \) is a language. Game:

- Alice is Poly time and she has \( x, |x| = n \).
- Bob is all powerful and he has nothing.
- They cooperate to determine if \( x \in L \). Alice could just send Bob \( x \). That takes \( n \) bits.
Let $L$ be the set of all 3-colorable graphs (or any NPC graph problem). Note size is $O(n^2)$. Is there a protocol for Alice and Bob in $O(n^{2-\epsilon})$ bits for some $\epsilon > 0$?
Let $L$ be the set of all 3-colorable graphs (or any NPC graph problem). Note size is $O(n^2)$. Is there a protocol for Alice and Bob in $O(n^{2-\epsilon})$ bits for some $\epsilon > 0$?

Dell and van Melkebeek showed that if there is a protocol in $O(n^{2-\epsilon})$ bits then the Poly Hierarchy Collapses to $\Sigma^p_2$. The proof used large 3-free set. [https://www.cs.umd.edu/users/gasarch/TOPICS/ramsey/dell.pdf](https://www.cs.umd.edu/users/gasarch/TOPICS/ramsey/dell.pdf)
Let $L$ be the set of all 3-colorable graphs (or any NPC graph problem). Note size is $O(n^2)$. Is there a protocol for Alice and Bob in $O(n^{2-\epsilon})$ bits for some $\epsilon > 0$?

- Dell and van Melkebeek showed that if there is a protocol in $O(n^{2-\epsilon})$ bits then the Poly Hierarchy Collapses to $\Sigma^p_2$. The proof used large 3-free set. [https://www.cs.umd.edu/users/gasarch/TOPICS/ramsey/dell.pdf](https://www.cs.umd.edu/users/gasarch/TOPICS/ramsey/dell.pdf)

- Too much prerequisite knowledge.
Hindman’s Thm

Hindman’s Thm For any finite coloring of \( \mathbb{N} \) there exists an infinite \( A \) such that all finite sums of elements of \( A \) are the same color.
Hindman’s Thm

Hindman’s Thm For any finite coloring of $\mathbb{N}$ there exists an infinite $A$ such that all finite sums of elements of $A$ are the same color.

- Proof use ultrafilters, so hard, but nowhere near as hard as Szemeredi’s Result.
Hindman’s Thm

For any finite coloring of $\mathbb{N}$ there exists an infinite $A$ such that all finite sums of elements of $A$ are the same color.

- Proof use ultrafilters, so hard, but nowhere near as hard as Szemeredi’s Result.
- I would need to brush up on this one (this is a PRO)
**Hindman’s Thm** For any finite coloring of \( \mathbb{N} \) there exists an infinite \( A \) such that all finite sums of elements of \( A \) are the same color.

- Proof use ultrafilters, so hard, but nowhere near as hard as Szemeredi’s Result.
- I would need to brush up on this one (this is a PRO)
- [https://web.williams.edu/Mathematics/lg5/Hindman.pdf](https://web.williams.edu/Mathematics/lg5/Hindman.pdf)
Hindman’s Thm

Hindman’s Thm For any finite coloring of \( \mathbb{N} \) there exists an infinite \( A \) such that all finite sums of elements of \( A \) are the same color.

- Proof use ultrafilters, so hard, but nowhere near as hard as Szemeredi’s Result.
- I would need to brush up on this one (this is a PRO)
- [https://web.williams.edu/Mathematics/lg5/Hindman.pdf](https://web.williams.edu/Mathematics/lg5/Hindman.pdf)
- **Research** Come up with an elementary proof.
Results Bill Likes But Would be Hard to Teach: Ramsey

Exposition by William Gasarch

May 10, 2022
Results from Logic

Thm
For every computable \( a \)-ary \( \mathcal{C} \):
\[
\text{(N) } a \to \left[ c \right]
\]
there is a \( \Pi a \)-homogenous set. There is a computable coloring such that no homog set is \( \Sigma a \).

▶ Too much prerequisite knowledge needed.
▶ Proof leads to a different proof of Ramsey's Thm!
▶ Important: measures how nonconstrutive the proof of Ramsey's Thm.
▶ Part of Recursive Combinatorics. My survey:

Thm
The full \( a \)-ary \( c \)-color Large Ramsey Thm cannot be proven from Peano Arithmetic.

▶ The first natural Thm to be shown ind. of PA.
▶ Too much prerequisite knowledge needed.
Results from Logic

Thm For every computable COL: $\binom{N}{a} \rightarrow [c]$ there is a $\Pi_a$-homogenous set. There is a computable coloring such that no homog set is $\Sigma_a$. 
Thm For every computable COL: \( (\mathbb{N})^a \rightarrow [c] \) there is a \( \Pi_a \)-homogenous set. There is a computable coloring such that no homog set is \( \Sigma_a \).

▷ Too much prerequisite knowledge needed.

Thm The full \( a \)-ary \( c \)-color Large Ramsey Thm cannot be proven from Peano Arithmetic.

▷ The first natural Thm to be shown ind. of PA.

▷ Too much prerequisite knowledge needed.
**Thm** For every computable $\text{COL}: (\mathbb{N}) \rightarrow [c]$ there is a $\Pi_a$-homogenous set. There is a computable coloring such that no homog set is $\Sigma_a$.

- Too much prerequisite knowledge needed.
- Proof leads to a different proof of Ramsey’s Thm!
Results from Logic

**Thm** For every computable $\text{COL}: \binom{\mathbb{N}}{a} \to [c]$ there is a $\Pi_a$-homogenous set. There is a computable coloring such that no homog set is $\Sigma_a$.

- Too much prerequisite knowledge needed.
- Proof leads to a different proof of Ramsey’s Thm!
- Important: measures how nonconstrutive the proof of Ramsey’s Thm.
Results from Logic

**Thm** For every computable \( \text{COL}: \binom{\mathbb{N}}{a} \to [c] \) there is a \( \Pi_a \)-homogenous set. There is a computable coloring such that no homog set is \( \Sigma_a \).

- Too much prerequisite knowledge needed.
- Proof leads to a different proof of Ramsey’s Thm!
- Important: measures how nonconstrutive the proof of Ramsey’s Thm.
**Thm** For every computable \( \text{COL}: (\mathbb{N}^a) \rightarrow [c] \) there is a \( \Pi^a \)-homogenous set. There is a computable coloring such that no homog set is \( \Sigma^a \).

- Too much prerequisite knowledge needed.
- Proof leads to a different proof of Ramsey’s Thm!
- Important: measures how nonconstructive the proof of Ramsey’s Thm.
- Part of **Recursive Combinatorics**. My survey:

**Thm** The full \( a \)-ary \( c \)-color Large Ramsey Thm cannot be proven from Peano Arithmetic.
**Thm** For every computable \( \text{COL}: \binom{\mathbb{N}}{a} \rightarrow [c] \) there is a \( \Pi^a \)-homogenous set. There is a computable coloring such that no homog set is \( \Sigma^a \).

- Too much prerequisite knowledge needed.
- Proof leads to a different proof of Ramsey’s Thm!
- Important: measures how nonconstrutive the proof of Ramsey’s Thm.


**Thm** The full \( a \)-ary \( c \)-color Large Ramsey Thm cannot be proven from Peano Arithmetic.

- The first natural Thm to be shown ind. of PA.
Results from Logic

**Thm** For every computable \( \text{COL}: \binom{\mathbb{N}}{a} \rightarrow [c] \) there is a \( \Pi_a \)-homogenous set. There is a computable coloring such that no homog set is \( \Sigma_a \).

▶ Too much prerequisite knowledge needed.
▶ Proof leads to a different proof of Ramsey’s Thm!
▶ Important: measures how nonconstrustive the proof of Ramsey’s Thm.
▶ Part of **Recursive Combinatorics**. My survey:

**Thm** The full \( a \)-ary \( c \)-color Large Ramsey Thm cannot be proven from Peano Arithmetic.

▶ The first natural Thm to be shown ind. of PA.
▶ Too much prerequisite knowledge needed.
Ramsey Multiplicity

Recall:

**Thm** For all 2-col of $K_n$, exists $\frac{n^3}{24} - O(n^2)$ mono $K_3$'s.
Recall:

**Thm** For all 2-col of $K_n$, exists $\frac{n^3}{24} - O(n^2)$ mono $K_3$'s.

This is the first thm in a field called **Ramsey Multiplicity**
Recall:

**Thm** For all 2-col of $K_n$, exists $\frac{n^3}{24} - O(n^2)$ mono $K_3$'s.

This is the first thm in a field called **Ramsey Multiplicity**

Here is the second thm
Recall:

**Thm** For all 2-col of $K_n$, exists $\frac{n^3}{24} - O(n^2)$ mono $K_3$'s.

This is the first thm in a field called **Ramsey Multiplicity**

Here is the second thm

**Thm** Fix $k$. For large $n$, for all 2-colorings of $K_n$ there exists

$$\frac{n^2}{4k^2(1+o(1))}$$

mono $K_k$'s.
Recall:

**Thm** For all 2-col of $K_n$, exists $\frac{n^3}{24} - O(n^2)$ mono $K_3$'s.

This is the first thm in a field called **Ramsey Multiplicity**

Here is the second thm

**Thm** Fix $k$. For large $n$, for all 2-colorings of $K_n$ there exists $\frac{n^2}{4k^2(1+o(1))}$ mono $K_k$'s.

This result was deemed too trivial to actually write up until I wrote it up (with help from Nathan Grammel and Erik Metz) in an open problems column on this topic https://www.cs.umd.edu/users/gasarch/open/Ramseymult.pdf
Ramsey Multiplicity

Recall:

**Thm** For all 2-col of $K_n$, exists $\frac{n^3}{24} - O(n^2)$ mono $K_3$'s.

This is the first thm in a field called **Ramsey Multiplicity**

Here is the second thm

**Thm** Fix $k$. For large $n$, for all 2-colorings of $K_n$ there exists $\frac{n^2}{4k^2(1+o(1))}$ mono $K_k$'s.

This result was deemed too trivial to actually write up until I wrote it up (with help from Nathan Grammel and Erik Metz) in an open problems column on this topic [https://www.cs.umd.edu/users/gasarch/open/Ramseymult.pdf](https://www.cs.umd.edu/users/gasarch/open/Ramseymult.pdf)

- Could teach this thm next time.
Ramsey Multiplicity

Recall:

**Thm** For all 2-col of $K_n$, exists $\frac{n^3}{24} - O(n^2)$ mono $K_3$'s.

This is the first thm in a field called **Ramsey Multiplicity**

Here is the second thm

**Thm** Fix $k$. For large $n$, for all 2-colorings of $K_n$ there exists $\frac{n^2}{4k^2(1+o(1))}$ mono $K_k$'s.

This result was deemed too trivial to actually write up until I wrote it up (with help from Nathan Grammel and Erik Metz) in an open problems column on this topic [https://www.cs.umd.edu/users/gasarch/open/Ramseymult.pdf](https://www.cs.umd.edu/users/gasarch/open/Ramseymult.pdf)

- Could teach this thm next time.
- **Research** Look at col $G$ to get mono $H$ for other $G$ and $H$. 
Results Bill Likes But Would be Hard to Teach: Complexity

Exposition by William Gasarch

May 10, 2022
Def $G \rightarrow (H_1, H_2)$ means that for every 2-coloring of the edges of $G$ there is either a RED $H_1$ or a BLUE $H_2$. 
Complexity: $\Pi^p_2$ Completeness of Arrow

Def $G \rightarrow (H_1, H_2)$ means that for every 2-coloring of the edges of $G$ there is either a RED $H_1$ or a BLUE $H_2$.

Marcus Schaefer proved the following.

Thm $\{ (G, H_1, H_2) : G \rightarrow (H_1, H_2) \}$ is $\Pi^p_2$-complete.

Grid Color Extension (GCE) is the set of tuples \((n, m, c, \chi)\) such that the following hold:

- \(n, m, c \in \mathbb{N}\). \(\chi\) is a partial \(c\)-coloring of \([n] \times [m]\) that is rectangle-free.
- \(\chi\) can be extended to a rectangle-free coloring of \([n] \times [m]\).
Grid Color Extension (GCE) is the set of tuples \((n, m, c, \chi)\) such that the following hold:

- \(n, m, c \in \mathbb{N}\). \(\chi\) is a partial \(c\)-coloring of \([n] \times [m]\) that is rectangle-free.
- \(\chi\) can be extended to a rectangle-free coloring of \([n] \times [m]\).

**Thm** (Apon, Gasarch, Lawler) GCE is NP-complete

Complexity: NP-Completeness of Grid Extension

*Grid Color Extension (GCE)* is the set of tuples \((n, m, c, \chi)\) such that the following hold:

- \(n, m, c \in \mathbb{N}\). \(\chi\) is a partial \(c\)-coloring of \([n] \times [m]\) that is rectangle-free.
- \(\chi\) can be extended to a rectangle-free coloring of \([n] \times [m]\).

**Thm** (Apon, Gasarch, Lawler) *GCE* is NP-complete


Jacob Proofread This!
Def Resolution proofs are a proof system to show that a Boolean Formula is NOT satisfiable. It is of interest to find a class of non-satisfiable formulas $\phi_n$ that require (say) $(1.5)^n$ long Res Proofs.
Def Resolution proofs are a proof system to show that a Boolean Formula is NOT satisfiable. It is of interest to find a class of non-satisfiable formulas \( \phi_n \) that require (say) \((1.5)^n\) long Res Proofs.

Def A graph is \( c \)-random if it does not contain a clique or ind set of size \( c \log n \).

Def \( \phi_{n,c} \) is a Boolean Formula that says every graph on \( n \) vertices is \( c \)-random. (This is false for \( c \) around \( \frac{1}{2} \).)
Def Resolution proofs are a proof system to show that a Boolean Formula is NOT satisfiable. It is of interest to find a class of non-satisfiable formulas $\phi_n$ that require (say) $(1.5)^n$ long Res Proofs.

Def A graph is $c$-random if it does not contain a clique or ind set of size $c \log n$.

Def $\phi_{n,c}$ is a Boolean Formula that says every graph on $n$ vertices is $c$-random. (This is false for $c$ around $\frac{1}{2}$.)

Lauria, Pudlak, Rodl, Thapen proved:

Thm For appropriate $c$, any resolution proof for $\phi_{n,c}$ requires length $n^{\Omega(\log n)}$.

I will let you decide which are PROS and which are CONS.
I will let you decide which are PROS and which are CONS.

- Students get to (or have to) learn some complexity theory.
PROS and CONS of Complexity of Ramsey

I will let you decide which are PROS and which are CONS.

- Students get to (or have to) learn some complexity theory.
- These results are Theoretical Computer science.
PROS and CONS of Complexity of Ramsey

I will let you decide which are PROS and which are CONS.

- Students get to (or have to) learn some complexity theory.
- These results are Theoretical Computer science.
- These results show that TCS=Math.
I will let you decide which are PROS and which are CONS.

- Students get to (or have to) learn some complexity theory.
- These results are Theoretical Computer science.
- These results show that TCS=Math.
- Bill would have to learn these proofs.
I will let you decide which are PROS and which are CONS.

- Students get to (or have to) learn some complexity theory.
- These results are Theoretical Computer science.
- These results show that TCS=Math.
- Bill would have to learn these proofs.
- Would take time away from more proofs of Can Ramsey.
PROS and CONS of Complexity of Ramsey

I will let you decide which are PROS and which are CONS.

▶ Students get to (or have to) learn some complexity theory.
▶ These results are Theoretical Computer science.
▶ These results show that TCS=Math.
▶ Bill would have to learn these proofs.
▶ Would take time away from more proofs of Can Ramsey.

Research What we really want is evidence that computing $R(k)$ is hard. These results do not really do that. Maybe you can!

Research Look at the above results for particular cases and see if easier.
Results Bill Does Not Care About But Should: VDW

Exposition by William Gasarch

May 10, 2022
Rado’s Thm for Matrices

Rado’s Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the $a_i$’s sums to 0.
- For all $c$, for all $\text{COL}: \mathbb{N} \to [c]$ there exists mono solution to

$$x_1 a_1 + \cdots + a_k x_k = 0.$$
Rado’s Thm for Matrices

Rado’s Thm
Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the $a_i$’s sums to 0.
- For all $c$, for all $COL: \mathbb{N} \rightarrow [c]$ there exists mono solution to

$$x_1a_1 + \cdots + a_kx_k = 0.$$

There is a version for systems of linear equations.
Rado’s Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the $a_i$’s sums to 0.
- For all $c$, for all $\text{COL}: \mathbb{N} \rightarrow [c]$ there exists mono solution to

$$x_1 a_1 + \cdots + a_k x_k = 0.$$ 

There is a version for systems of linear equations.

It’s a real pain to state and I don’t care.
Rado’s Thm Let $a_1, \ldots, a_k \in \mathbb{Z}$. TFAE

- Some subset of the $a_i$’s sums to 0.
- For all $c$, for all $\text{COL}: \mathbb{N} \rightarrow [c]$ there exists mono solution to

$$x_1 a_1 + \cdots + a_k x_k = 0.$$ 

There is a version for systems of linear equations. It’s a real pain to state and I don’t care.

For a statement of the thm see the Wikipedia entry.
Hales-Jewitt Thm

A Very General Thm from which we can derive cleanly VDW and Gallai-Witt.
Hales-Jewitt Thm

A Very General Thm from which we can derive cleanly VDW and Gallai-Witt.

Too abstract for my tastes, but its very important!
Hales-Jewitt Thm

A Very General Thm from which we can derive cleanly VDW and Gallai-Witt.

Too abstract for my tastes, but its very important!

See Wikipedia Entry for Statement.
Hales-Jewitt Thm

A Very General Thm from which we can derive cleanly VDW and Gallai-Witt.

Too abstract for my tastes, but its very important!

See Wikipedia Entry for Statement.

This is someone else’s slides on it. So I REALLY could have covered it!

Ramsey’s thm for n-parameter sets

Too complicated to state.
Ramsey’s thm for n-parameter sets

Too complicated to state.

Can derive Ramsey’s Thm and the Hales-Jewitt Thm from it.
Ramsey’s thm for n-parameter sets

Too complicated to state.

Can derive Ramsey’s Thm and the Hales-Jewitt Thm from it.

Results Bill Does Not Care About But Should: Ramsey

Exposition by William Gasarch

May 10, 2022
Thm (AC) There is a coloring of $\binom{\mathbb{R}}{2}$ with no homog set of size $\mathbb{R}$. 
**Ramsey Over the Reals Fails: So what to do?**

**Thm (AC)** There is a coloring of \( (\mathbb{R})_2 \) with no homog set of size \( \mathbb{R} \).

So what to do?
Ramsey Over the Reals Fails: So what to do?

**Thm** (AC) There is a coloring of $\left(\mathbb{R}^2\right)$ with no homog set of size $\mathbb{R}$. So what to do?

- **Research Topic** Assume $\neg AC$ and perhaps something else like $AD$. 
Ramsey Over the Reals Fails: So what to do?

**Thm** (AC) There is a coloring of $\left(\mathbb{R}^2\right)$ with no homog set of size $\mathbb{R}$. So what to do?

- **Research Topic** Assume $\neg AC$ and perhaps something else like $AD$.
Thm (AC) There is a coloring of $\binom{\mathbb{R}}{2}$ with no homog set of size $\mathbb{R}$.

So what to do?

▶ **Research Topic** Assume $\neg AC$ and perhaps something else like $AD$.


▶ Prove what you can: If $\kappa$ is a cardinal then for all $\text{COL}: \binom{2^{\kappa+}}{2}$ there is a homog set of size $\kappa$. 

Ramsey Cardinals on Next Slide.
Ramsey Over the Reals Fails: So what to do?

**Thm** (AC) There is a coloring of $\binom{\mathbb{R}}{2}$ with no homog set of size $\mathbb{R}$.

So what to do?

- **Research Topic** Assume $\neg AC$ and perhaps something else like $AD$.


- Prove what you can: If $\kappa$ is a cardinal then for all $\text{COL}: \binom{2\kappa^+}{2}$ there is a homog set of size $\kappa$.

- Ramsey Cardinals on Next Slide.
Inaccessible Cardinals

True and Obvious If $\alpha < \aleph_0$ then $2^\alpha < \aleph_0$.
Inaccessible Cardinals

**True and Obvious** If \( \alpha < \aleph_0 \) then \( 2^\alpha < \aleph_0 \).

**Question** Does there exist an infinite \( \kappa > \aleph_0 \) such that if \( \alpha < \kappa \) then \( 2^\alpha < \kappa \).
Inaccessible Cardinals

**True and Obvious** If $\alpha < \aleph_0$ then $2^{\alpha} < \aleph_0$.

**Question** Does there exist an infinite $\kappa > \aleph_0$ such that if $\alpha < \kappa$ then $2^\alpha < \kappa$.

**Vote**: YES, NO, or OTHER.
True and Obvious If \( \alpha < \aleph_0 \) then \( 2^\alpha < \aleph_0 \).

Question Does there exist an infinite \( \kappa > \aleph_0 \) such that if \( \alpha < \kappa \) then \( 2^\alpha < \kappa \).

Vote: YES, NO, or OTHER.

OTHER.
Inaccessible Cardinals

**True and Obvious** If $\alpha < \aleph_0$ then $2^\alpha < \aleph_0$.

**Question** Does there exist an infinite $\kappa > \aleph_0$ such that if $\alpha < \kappa$ then $2^\alpha < \kappa$?

**Vote**: YES, NO, or OTHER.

OTHER. I know what you are thinking.
Inaccessible Cardinals

**True and Obvious** If $\alpha < \aleph_0$ then $2^\alpha < \aleph_0$.

**Question** Does there exist an infinite $\kappa > \aleph_0$ such that if $\alpha < \kappa$ then $2^\alpha < \kappa$.

**Vote:** YES, NO, or OTHER.

OTHER. I know what you are thinking. Ind of ZFC.
Inaccessible Cardinals

True and Obvious If $\alpha < \aleph_0$ then $2^\alpha < \aleph_0$.

Question Does there exist an infinite $\kappa > \aleph_0$ such that
If $\alpha < \kappa$ then $2^\alpha < \kappa$.

Vote: YES, NO, or OTHER.

OTHER. I know what you are thinking. Ind of ZFC. Not quite.
Inaccessible Cardinals

**True and Obvious** If $\alpha < \aleph_0$ then $2^{\alpha} < \aleph_0$.

**Question** Does there exist an infinite $\kappa > \aleph_0$ such that if $\alpha < \kappa$ then $2^{\alpha} < \kappa$.

**Vote**: YES, NO, or OTHER.

OTHER. I know what you are thinking. Ind of ZFC. Not quite.

**Thm** ZFC cannot prove that such a $\kappa$ exists.

**Pf** Such a $\kappa$ would be a model of ZFC. No theory can prove the existence of a model for itself.
Inaccessible Cardinals

**True and Obvious** If $\alpha < \aleph_0$ then $2^\alpha < \aleph_0$.

**Question** Does there exist an infinite $\kappa > \aleph_0$ such that if $\alpha < \kappa$ then $2^\alpha < \kappa$.

**Vote:** YES, NO, or OTHER.

OTHER. I know what you are thinking. Ind of ZFC. Not quite.

**Thm** ZFC cannot prove that such a $\kappa$ exists.

**Pf** Such a $\kappa$ would be a model of ZFC. No theory can prove the existence of a model for itself.

**What About...** Can ZFC prove that such a $\kappa$ does not exist?
Inaccessible Cardinals

**True and Obvious** If $\alpha < \aleph_0$ then $2^\alpha < \aleph_0$.

**Question** Does there exist an infinite $\kappa > \aleph_0$ such that if $\alpha < \kappa$ then $2^\alpha < \kappa$.

**Vote:** YES, NO, or OTHER.

OTHER. I know what you are thinking. Ind of ZFC. Not quite.

**Thm** ZFC cannot prove that such a $\kappa$ exists.

**Pf** Such a $\kappa$ would be a model of ZFC. No theory can prove the existence of a model for itself.

**What About**... Can ZFC prove that such a $\kappa$ does not exist? Unknown.
**Inaccessible Cardinals**

**True and Obvious** If $\alpha < \aleph_0$ then $2^\alpha < \aleph_0$.

**Question** Does there exist an infinite $\kappa > \aleph_0$ such that if $\alpha < \kappa$ then $2^\alpha < \kappa$.

**Vote**: YES, NO, or OTHER.

OTHER. I know what you are thinking. Ind of ZFC. Not quite.

**Thm** ZFC cannot prove that such a $\kappa$ exists.

**Pf** Such a $\kappa$ would be a model of ZFC. No theory can prove the existence of a model for itself.

**What About**... Can ZFC prove that such a $\kappa$ does not exist? Unknown.

**Def** $\kappa$ is **inaccessible** if $\alpha < \kappa \implies 2^\alpha < \kappa$. 


Ramsey Cardinals

**Def** If for all $\text{COL}: \binom{\kappa}{2}$ there is a homog set of size $\kappa$ then $\kappa$ is **Ramsey**.

**True** $\aleph_0$ is Ramsey.

**Question** Does there exist a Ramsey cardinal $\kappa > \aleph_0$?

**Vote**: YES, NO, or OTHER.

**Thm** If $\kappa$ is Ramsey then $\kappa$ is inaccessible. (The converse is ind of ZFC but reasons to think its false.)
**Defs**: If for all COL: \( \binom{\kappa}{2} \) there is a homogen set of size \( \kappa \) then \( \kappa \) is **Ramsey**.

**True** \( \aleph_0 \) is Ramsey.

**Question** Does there exist a Ramsey cardinal \( \kappa > \aleph_0 \) ?

**Vote**: YES, NO, or OTHER.
Ramsey Cardinals

**Def** If for all COL: \( (\kappa^2) \) there is a homog set of size \( \kappa \) then \( \kappa \) is Ramsey.

**True** \( \aleph_0 \) is Ramsey.

**Question** Does there exist a Ramsey cardinal \( \kappa > \aleph_0 \)?

**Vote:** YES, NO, or OTHER.

**Thm** If \( \kappa \) is Ramsey then \( \kappa \) is inaccessible. (The converse is ind of ZFC but reasons to think its false.)
Results Bill May One Day Learn But Still too Hard for the Students

Exposition by William Gasarch

May 10, 2022
Ramsey’s Thm with control of the differences

**Thm** For all $c, k$ and for all order types $\eta$ there exists $N = N(c)$ such that for all $\text{COL}: [N] \to [c]$ there exists a homog set $a_1 < \cdots < a_k$ such that

$$(a_2 - a_1, a_3 - a_2, \ldots, a_n - a_{n-1})$$

are all distinct and are in order type $\eta$.

- First proven by Noga Alon and Jan Pach using VDW, so bounds on $N(c)$ are large. Later Noga Alon, Alan Stacey, and Saharon Shelah got an iterated exp bound. None of this is written down anywhere.


- Shelah’s paper is hard. I’m looking for easier proof of weaker results.
Szemeredi, Furstenberg, Gowers have given different proofs of:

**Sz Thm** If $A$ has upper pos density then, for all $k$, $A$ contains a $k$-AP.
Szemeredi, Furstenberg, Gowers have given different proofs of:

**Sz Thm** If $A$ has upper pos density then, for all $k$, $A$ contains a $k$-AP.

All the proofs look hard to learn and to teach.
Szemeredi, Furstenberg, Gowers have given different proofs of:

**Sz Thm** If $A$ has upper pos density then, for all $k$, $A$ contains a $k$-AP.

All the proofs look hard to learn and to teach.

**Research** Easier Proof.

Caveat
There is a proof of Sz thm for Hales-Jewitt which is said to be elementary.

https://arxiv.org/abs/0910.3926
Szemeredi, Furstenberg, Gowers have given different proofs of:

**Sz Thm** If $A$ has upper pos density then, for all $k$, $A$ contains a $k$-AP.

All the proofs look hard to learn and to teach.

**Research** Easier Proof.

**Caveat** There is a proof of Sz thm for Hales-Jewitt which is said to be elementary.

https://arxiv.org/abs/0910.3926
Thm For all $k$ the set of primes has a $k$-AP.
Theorem For all $k$ the set of primes has a $k$-AP.
Seems hard to learn and teach.
Green-Tao Thm

**Thm** For all $k$ the set of primes has a $k$-AP.

Seems hard to learn and teach.

**Research** Easier proof, perhaps of subcases.
Green-Tao Thm

**Thm** For all $k$ the set of primes has a $k$-AP.

Seems hard to learn and teach.

**Research** Easier proof, perhaps of subcases.

**Research** Look for the AP’s.