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Project Re-derive the known obs sets with a SAT Solver and (try to) find the obs set for 5-coloring.

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Look at avoidance game- try to NOT get K_3.

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Caveat I've had some HS students do this before for Graphs. Might want to do it for VDW's Theorem, Poly VDW, Square-Thm, Grid-Thms.

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See next page for variants.

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Combinations of the above. Empirical studies of the above.

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Projects Good writeup of the known proof(s) of this, perhaps better bound on the numbers. Possibly empirical results.

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Project Read, undestand, and write up these results. Then see if you can extend.

Euclidean Ramsey

See next lecture