752 Final for Spring 2026

Exam START TIME: 12:00PM (Noon) May 15

Exam END TIME: 6:00PM May 15

My Expectation: The exam should take between 2 and 3 hours, but feel free to take all 6.

Rules:

- You CAN use your slides, notes, HW, and your brain.
- You CAN ask me a question of clarity by email. I may also be on zoom part of the day.
- You CANNOT use anything else.
- How will I enforce the rules? I am counting on your integrity and fine character.
- The exam is one 0-point problem (what is your name) and then four 20-point problems. So the totals to 80 points. The other 20 points are from the take-home part of the final.
- Exam starts on the next page.

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1. (0 points) What is your name

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2. If X is a finite subset of N then $\min(X)$ is the smallest element of X.

   Recall that ACK is a very fast growing function of two variables.

   We want a fast growing function on one variable.

   Let $f(n) = \mathrm{ACK}(n, n)$.

   Note that f is a fast growing function on one variable.

   Def Let X be a finite subset of N. X is *superlarge* if $|X| > f(\min(X))$.

   Prove the following:

   For every $k \in \mathsf{N}$ *there exists* n *such that,*

   for every $\mathrm{COL}\colon \binom{k,k+1,\ldots,k+n}{3} \rightarrow [2]$ *there exists a superlarge homog set.*

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3. (20 points) Recall that infinite a-ary Ramsey Theorem:

   For all $a \in \mathsf{N}$, *for all* $\mathrm{COL}\colon \binom{\mathsf{N}}{a} \rightarrow [2]$ *there exists an infinite homog set.*

   We denote the above statement as a-ary IRT (Infinite Ramsey Theorem).

   Assume that you already know the 1-ary IRT, 2-ary IRT, ..., 99-ary IRT.

   Prove the 100-ary IRT.

   (Hint: There are many different proofs depending on which a-IRT you want to use infinitely often and which one at the end. Do the easiest one which is NOT the one that gives the best bounds when made finite.)

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4. (20 points) Let E be the equation

$$x_1 + 2x_2 + 4x_3 - 8x_4 = 0$$

Give a finite coloring of N that has no mono solution in N. Prove your coloring has no mono solution. (For this problem 0 is NOT an element of N.)

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5. (20 points) Recall that $W(k,2)$ is

   the least n *such that, for all* COL: $[n]\to[2]$, *there is a mono* k*-AP.*

   Use the probabilistic method to obtain a lower bound on $W(k,2)$.

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