Homework 03, Morally Due 12:30PM, Tue Feb 17, 2026

1. (0 points) What is your name?

2. (20 points)

   (a) (0 points)

   Look at the slide packet *Infinite Ramsey Thm For Graphs.*

   Look at Slides 102-119.

   Slide 102 is titled *Formal Construction.*

   Slide 119 is titled *Some Color Appears Infinitely Often.*

   READ and UNDERSTAND the construction and how it captures the intuitive construction I presented in that slide packet.

   (b) (20 points)

   Look at the slide packet *Infinite Ram on Z*

   Look at Slides 70-100.

   Slide 70 is titled $c \leq 2$

   Slide 100 is titled *We Are Done*

   Write up the construction formally, similar to what you read and understood in Part 1 of this question.

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3. (20 point) Look at the slide packet *Inf Can Ram*
   Look at Slides 87–94, the first one titled *Step 3*

   (a) (10 points) Describe how to process x_3.
   (b) (10 points) Describe how to process x_n.

4. (20 point) Look at the slide packet *Inf Can Ram*.

   Look at Slide 139-149. The first one is titled *Is H_1 Max-Homog?*

   Prove that H_1 is Max-Homog.

5. (20 points) Look at the slide packet *Inf Can Ram.*

   Look at Slides 168–177. The first one is titled *Need to do More To Get Rainbow Set.*

   Prove the theorem on slide 177:

   Let X be infinite. Let $\mathrm{COL}\colon \binom{X}{2} \to \omega$. *Assume that, for all* $c \in \omega$, $\deg_c(x) \le 2$. *Then there exists an infinite rainbow* $X_1 \subseteq X$.

6. (20 points) Let $A = \{p_1, p_2, \ldots\}$ be a sequence of distinct points in the plane. Let $d(p_i, p_j)$ be the usual Euclidean distance from p_i to p_j.

   Let COL: $\binom{\mathsf{N}}{2} \to \mathsf{R}$ be the coloring

$$\mathrm{COL}(i, j) = d(x_i, x_j).$$

   Apply the Can Ramsey Theorem to this coloring to get an infinite set H.

   (a) (3 points) Prove that H is not homog.

   (b) (3 points) Prove that H is not min-homog.

   (c) (3 points) Prove that H is not max-homog.

   (d) (11 points) So H is Rainbow. You just proved a theorem! I'll begin the statement and then you need to FILL IN the part that says FILL IN.

   Let $A = \{p_1, p_2, \ldots\}$ be a sequence of distinct points in the plane. There is an infinite $B \subseteq A$ such that FILL IN.

   (e) (0 points and not extra credit, but for your own personal enlightenment) Think about the following variants of this problem:

   - Use a different definition of distance, E.g., *the Manhattan Metric (look it up).*
   - Do the problem in R^3 with both the Euclidean distance and others.
   - To quote *The Bolzano Weierstrass rap*:
     Well I am out of here, because my rap is at its end,
     But I'll leave you with this exercise, to do it in R^n!

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7. (0 points but you must do it)

   (a) Listen to as much of *The Bolzano Weierstrass Rap* as you can stand. Here it is:
   `https://www.youtube.com/watch?v=eM3S74kchoM&list=FLllaX0oj-4_KcE1C4JF72gw`

   (b) I feel a moral obligation to present you with math songs that are better than the BW Rap (hmmm, that would be all math songs).
   Here is a link to my collection of math songs:
   `https://www.cs.umd.edu/~gasarch/FUN/mathsongs.html`
   Pick 5 using any criteria you like (e.g., You are a Swiftie so you want to listen to the linear algebra song *u belongs to V*), listen to them, and give me your opinion of some of them as you see fit.

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8. Extra Credit (0 points). This problem does not use any of the techniques in the course. I will be asking this this to my Honors Discrete Math class and see who does better, them or you (I would bet on you).

   Def An *honest c-coloring* is a coloring that uses all c colors.

   Def Let COL: $\mathsf{R}^n \rightarrow [c]$. Let $b \le c$.

   A *b-rainbow line* is a line that has at least b colors on it.

   A *b-rainbow plane* is a plane that has at least b colors on it.

   (a) Let A be the set of all pairs (b, c) such that $b \le c$ and the following is true:

   For all honest c-colorings of R^2 *there is a b-rainbow line.*

   Give a simple description of the set A (e.g., *A is the set of all (b, c) such that* $1 \le b^2 \le c$). Prove the your descriptions is correct—that is, all (b, c) that fit your description have the property, and all (b, c) that satisfy the property fit your description.

   (b) Let B be the set of all pairs (b, c) such that $b \le c$ and the following is true:

   For all honest c-colorings of R^3 *there is a b-rainbow line.*

   Give a simple description of the set B, for example

   B is the set $\{(1, 2), (1, 3), (2, 3)\}$

   Prove the your descriptions is correct—that is, all (b, c) that fit your description have the property, and all (b, c) that satisfy the property fit your description.

   (c) Let C be the set of all ordered pairs (b, c) such that $b \le c$ and the following is true:

   For all honest c-colorings of R^3 *there is a b-rainbow plane.*

   Give a simple description of the set C, for example

   C is the set of all (b, c) such that b is prime and c is the product of two primes

   Prove the your descriptions is correct—that is, all (b, c) that fit your description have the property, and all (b, c) that satisfy the property fit your description.