

Homework 05, Morally Due 12:30PM, Tue Mar 03 2026

1. (0 points) What is your name?

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2. (35 points) Let $\Sigma = \{a, b\}$. Let Σ^* be the set of all strings over Σ including the empty string (Example: $aabba$, $bababba$.)

If $x, y \in \Sigma^*$ then x is a *subsequence of y* if you can take y , remove some letters, and get x .

Example: $abba$ is a subsequence of $aababaabaaaabaa$

We define $x \preceq y$ to mean that x is a subsequence of y .

Show that (Σ^*, \preceq) is a well quasi order by using a minimal bad sequence argument.

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3. (0 point but you must do this to appreciate what I tell you when I go over the subsequent problem in class. Do not hand anything in for this problem.)

If $w \in \Sigma^*$ then $\text{SUBSEQ}(w)$ is the set of all subsequences of w .

Let $L \subseteq \{a, b\}^*$. Then $\text{SUBSEQ}(L) = \cup_{w \in L} \text{SUBSEQ}(w)$.

- (a) Look at these two slide packets from my CMSC 452, on regular languages.

<https://www.cs.umd.edu/~gasarch/COURSES/452/S25/slides/dfatalk.pdf>

<https://www.cs.umd.edu/~gasarch/COURSES/452/S25/slides/nfatalk.pdf>

Using the DFA NFA equivalence show the following:

If L is regular then $\text{SUBSEQ}(L)$ is regular.

- (b) Look at this two slide packets from my CMSC 452, on context free languages. (Only need up to around slide 23—the definitions of a context free grammar and of a context free language.)

<https://www.cs.umd.edu/~gasarch/COURSES/452/S25/slides/cfgtalk.pdf>

Using the definition of context-free grammar show the following:

If L is a context-free language then $\text{SUBSEQ}(L)$ is a context-free language.

- (c) I assume you all know what P is, the set of languages in polynomial time.

Is the following TRUE, FALSE, or UNKNOWN TO SCIENCE:

If L is in P then $\text{SUBSEQ}(L)$ is in P .

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4. (35 points) For this problem I define the minor ordering on colored graphs. $H \preceq_{c-m} G$, as follows: $H \preceq_{c-m} G$ if you can take G and, by a sequence of the following operations, obtain H : (a) contract an edge and choose one of the colors of the endpoints to be the color of the merged vertex, (b) remove a vertex, (c) remove an edge.

For this problem you can assume the GMT for COLORED graphs:

Let $n \in \mathbb{N}$. If G_1, G_2, \dots , is an infinite sequence of n -colored graphs then there exists $i < j$ such that $G_i \preceq_{c-m} G_j$.

Let $\text{GM}(n)$ be the largest number such that there exists a bad sequence (using \preceq_{c-m}) of n -colored graphs

$G_1, G_2, \dots, G_{\text{GM}(n)}$

where G_i has at most i vertices.

Show that $\text{GM}(n)$ exists.

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5. (30 points) Fix $k \in \mathbb{N}$. Assume that, for all $1 \leq i \leq k$, WE HAVE an FPT algorithm for $VC_i = \{G: G \text{ has a vertex cover of size } \leq i\}$.

(Recall that this means we really have the code.)

Prove that WE HAVE an FPT algorithm for the following FUNCTION:

On input $G = (V, E)$:

- If $G \notin VC_k$ then output NO.
- If $G \in VC_k$ then output YES and ALSO output a vertex cover $U \subseteq V$ of size $\leq k$.

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6. (0 points. Do for your own enlightenment. DO NOT hand in.)

Let (X, \preceq) be a wqo. $2^{\text{fin}X}$ is the set of finite subsets of X .

We define the following ordering on $2^{\text{fin}X}$

$A \preceq_1 B$ if there exists a 1-1 function $f: A \rightarrow B$ such that $x \preceq f(x)$.

Note that \preceq_1 is defined using \preceq .

Show that $(2^{\text{fin}X}, \preceq_1)$ is a wqo.

Hint: This is an adjustment of the proof that $2^{\text{fin}\mathbb{N}}$ under a similar ordering is a wqo.

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7. (Extra Credit)

I first discuss the subtlety about the Kruskal Tree Theorem that I alluded to in class.

I defined the minor ordering, denoted $H \preceq_m G$, as follows: $H \preceq_m G$ if you can take G and, by a sequence of the following operations, obtain H : (a) contract an edge. (b) remove a vertex, (c) remove an edge.

Here is what I stated in class which, while true, is not what you should try to prove:

STATEMENT A: *The set of trees under \preceq_m is a wqo.*

DO NOT try to prove STATEMENT A. It is true; however, to prove it you need to prove something *stronger*. This is one of those cases where it is easier to prove a stronger theorem.

I now define the minor' ordering, denoted $H \preceq'_m G$, as follows: $H \preceq'_m G$ if you can take G and, by a sequence of the following operations, obtain H : (a) contract an edge. Oh. Just one operation.

Here is what you can prove similar to the other proofs:

STATEMENT B: *The set of trees under \preceq'_m is a wqo.*

For the extra credit prove STATEMENT B. ALSO (and this is probably the only part I will look at)

- (a) State clearly which part of the proof breaks down if you use \preceq_m instead of \preceq'_m .
- (b) State clearly which part of the proof breaks down if you use SUBGRAPH instead of \preceq'_m .