Homework 10, Morally Due 12:30PM, Tue Apr 21 2026

1. (0 points) What is your name?

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2. (35 points) For this problem assume $R(5) = 43$ (this is widely believed). Hence the following is true:

For all COL: $\binom{[43]}{2} \rightarrow [2]$, *there exists at least ONE mono* K_5.

(a) (20 points) Fill in the following sentence and prove it:
Let $L \in \mathsf{N}$. *Think of* L *as SMALL (like* $L = 2, 3, 4, 5$.*) If* n *satisfies condition* $XXX(L)$ *then, for all* COL: $\binom{[n]}{2} \rightarrow [2]$, *there exists at least* L *mono* K_5*'s. This should use a similar technique to what I did in class with* K_4. *(NOTE: DO NOT do the following: for* $L = 2$, *use* $n = 2 \times 43$, *for* $L = 3$, *use* $n = 3 \times 43$.*)*

(b) (0 point and don't hand in but this will help you with Part c) Write a program that will, given L, find the least n that satisfies $XXX(L)$.

(c) (15 points) Run the program for $L = 1$ to 20 and give us a table of the following form (I only give the first three rows

L	least n
1	43
2	50
3	57

(d) (0 points) Using the data from the last problem, try to get a function that approximates n as a function of L.

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3. (35 points)

Def Let X be a finite set. Let COL: $X \rightarrow \omega$. Let $Y \subseteq X$. Y is *homog* if all the elements of Y are the same color. Y is *rainbow* if all the elements of Y are different colors.

The following is the 1-ary Large Can Ramsey:

For all k there exists $n = LCR(k)$ such that,

for all COL: $\{k, k+1, \ldots, k+n\} \rightarrow \omega$ *either there exists a large homog set or a large rainbow set.*

(a) (35 points) Prove the 1-dim large can Ramsey theorem.

(b) (0 points but you must do) Does your prove give a bound on $LCR(k)$ (perhaps in terms of large Ramsey numbers of a higher arity).?

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4. (30 points) Let $a, b, c \geq 2$. Let $R(a, b, c)$ be the least n such that, for all COL: $\binom{[n]}{2} \to [3]$ there exists either a RED K_a or a BLUE K_b or a GREEN K_c. (So $R(k, k, k)$ is the Ramsey Number for 3-coloring a graph and looking seeking a homog set of size k.)

   (a) (0 points but you should do it) Show that
   $R(2, b, c) = R(b, c)$
   $R(a, 2, c) = R(a, c)$
   $R(a, b, 2) = R(a, b)$.

   (b) (20 points) Show that
   $R(a, b, c) \leq R(a-1, b, c) + R(a, b-1, c) + R(a, b, c-1) - 1$.

   (c) (0 points) Write a program that will, on input a, b, outputs an upper bound on $R(a, b)$. Use the recurrence-inequality we had for $R(a, b)$ and also the trick where if the bounds on $R(a, b-1)$ and $R(a-1, b)$ are even then the bound on $R(a, b)$ is $R(a-1, b) + R(a, b-1) - 1$. (We will be running it on small enough values that doing it by recursion will be fine.)

   (d) (0 points) Write a program that will, on input a, b, c, outputs an upper bound on $R(a, b, c)$. (We will be running it on small enough values that doing it by recursion will be fine. This program will use the $R(a, b)$ program.) (I do not think there is a trick to lower the number by 1.)

   (e) (10 points) Use your program to find upper bounds on $R(a, b, c)$ for $2 \leq a, b, c \leq 6$. DO NOT report all of these values. Since Megan ONLY cares about the symmetric case, JUST output the bounds on $R(2, 2, 2)$, $R(3, 3, 3)$, ... $R(8, 8, 8)$.
   So I want a table like the one below (the numbers are made up)

k	bound on $R(k, k, k)$
2	7
3	100
4	1000
5	2000
6	3000
7	4000
8	5000

   (f) (0 points) Using the data try to come up with a function f which is $R(k, k, k)$ as a function of k.