

**Homework 12, Morally Due 12:30PM, Tue May 5 2026**

1. (0 points) What is your name?

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2. (30 points)

- (a) (0 points) Write a program that, on input  $(n, k, c)$ , looks at ALL  $c$ -colorings of  $[n]$ , counts how many HAVE a  $k$ -AP, and divides that by  $c^n$ . Hence you get the fraction of  $c$ -colorings that have a mono  $k$ -AP to 3 places.
- (b) (15 points) For  $3 \leq n \leq 7$  determine what fraction of 2-colorings of  $[n]$  have a mono 3-AP. Output a table like the one below. THE NUMBERS WE GIVE ARE FALSE or true by accident.

$n$	Fraction of 2-colorings of $[n]$ that have a mono 3-AP
3	0.5
4	0.617
5	0.689
6	0.75
7	0.81
8	0.999

- (c) (15 points) For  $3 \leq n \leq 20$  determine what fraction of 2-colorings of  $[n]$  have a mono 4-AP. Output a table similar to the one in the last problem.
- (d) (0 points) Are the numbers surprising (e.g., more or less than you would have thought)?

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3. (40 points. This problem is also on the optional project so if you are doing the optional project, then use your answer for both this HW and the project. DO NOT say on the optional project *see my answer on HW12*).

Let  $VDW(k, \omega)$  mean that VDW theorem is known for  $k$ -APs and ANY number of colors.

Assume you know VDW theorem for  $VDW(1, \omega)$ ,  $VDW(2, \omega)$ ,  $VDW(3, \omega)$ , and  $VDW(4, \omega)$ . Prove VDW theorem for  $VDW(5, 2)$ .

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4. (30 points.)

You cannot Use Rado's Theorem for this problem. We will be asking you to prove Rado's Theorem in this particular case. You CAN use the Extended VDW theorem.

Consider the equation

$$E : v + 2w - 3x + y + 10z = 0.$$

Show that there exist a number  $M$  such that, for all  $\text{COL} : [M] \rightarrow [2]$ , there exists a mono solution to  $E$  (The  $v, w, x, y, z$  might not be distinct.)

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5. (0 points. AND YOU MUST DO THIS BY MONDAY SO THAT I CAN TALK ABOUT IT TUESDAY.)

What is funniest thing you saw in class. Here are options though you are not limited to them.

- (a) The Bolzano Weisterauss Rap
- (b) The repeated sketch where Ryan's Friend and I changed shirts.
- (c) The Banach Tarski Paradox used to explain the miracle of loaves and fishes.
- (d) The Combinatorics Song
- (e) Bill's Rap about the Infinite Hat Game.
- (f) BAD SEQUENCES! Song about the infinite hat game.
- (g) Bill sold the rights to the song to Who?
- (h) Celebrating Ryan's friend getting his Masters
- (i) *That is what Erdos calls it The Happy Ending Theorem.  
Their marriage lasted 68 years and they died on the same day.  
We prove this 3 ways.*
- (j) The Application of Ramsey Theory to History.
- (k) Quantum Supremacy vs you-know-what.

I am sure there are others. For example,

- (a) In the past one of the students said that the poly VDW theorem was funny since the bounds were so big. I didn't do TREE(3) that year which she would have thought was hilarious.
- (b) Some students have pointed to a line or part of a sketch.