Take Home Part of 752 Midterm for Spring 2026
Due Before Class March 31. NO EXTENSIONS

1. (0 points) What is your name

GO TO NEXT PAGE

2. (25 points)

For this problem you can use the following approximations to the truth.

Let $R_a(k)$ be the a-ary Ramsey numbers.

- $R_2(k) \le 2^{2k}$
- For all $\mathrm{COL}\colon \binom{[n]}{2} \to [2]$ there is a homog set of size $\ge \log n$.
- $R_3(k) \le 2^{2^{4k}}$.
- For all $\mathrm{COL}\colon \binom{[n]}{3} \to [2]$ there is a homog set of size $\ge \log \log n$.
- We denote the inverse of $\log^{(i)}(m)$ by $TOW_i(k)$.

Recall that statement of the 4-ary Ramsey Theorem.

Thm $(\forall k)$, there exists n such that, for all $\mathrm{COL}\colon \binom{[n]}{4} \to [2]$ there exists a homog set of size k.

We now (finally) state the problem.

There are three ways to prove the finite 4-ary Ramsey Theorem.

We state the three ways. For each way, prove the finite 4-ary Ramsey Theorem using that way, and give the bounds on n as a function of k that that way yields.

In your three proofs you should do the following:

- The first 3 steps of the construction.
- Stage s of the construction which yields a set H_s.
- A lower bound on $|H_s|$.
- What n needs be as a function of k.

Here are the 3 ways:

(a) Apply 3-ary finite Ramsey many times, then 1-ary Ramsey once.

(b) Apply 2-ary finite Ramsey many times, then 2-ary Ramsey once.

(c) Apply 1-ary finite Ramsey many times, then 3-ary Ramsey once.