

Homework 07, Morally Due 12:30PM, Tue Mar 24 2026

1. (0 points) What is your name?

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2. (35 points) Prove the infinite 4-ary Ramsey theorem:

For all COL: $\binom{\mathbb{N}}{4} \rightarrow [2]$ there exists an infinite homog set.

Do the proof where at every stage you use the infinite 3-ary Ramsey Theorem (so you use the infinite 3-ary Ramsey Theorem infinitely often) and then at the end you use the 1-ary Ramsey Theorem once.

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3. (30 points) Prove the finite 4-ary Ramsey theorem:

$\forall k)(\exists n)$ such that \forall COL: $\binom{[n]}{4} \rightarrow [2]$ there exists a homog set, size k .

Also give an upper bound on n as a function of k . The bound should be reasonable (e.g., NOT the TOWER function).

You may use the following approximation of the finite 3-ary Ramsey Theorem:

\forall COL: $\binom{[n]}{3} \rightarrow [2]$ there exists a homog set of size $\geq \log(\log(n))$.

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4. (35 points)

(a) Find XXX such that the following is true, and prove it.

Let $\text{COL}' : \binom{\mathbb{N}}{2} \times \mathbb{N} \rightarrow [10^{100}]$. Show that there exists infinite sets $A, B \subseteq \mathbb{N}$ such that COL' restricted to $\binom{A}{2} \times B$ only uses XXX colors.

(b) Find c such that the following is true, and prove it.

*Let $\text{COL}^{(\omega+\omega)} : \binom{\omega+\omega}{3} \rightarrow [10^{100}]$. Show that Then there exists an infinite c -homogenous set.
(Hint: Use Part a. Thats why its there!)*

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5. (0 points, but you must answer it)

As I told you in class, I wrote *I can see for miles and miles* as a song about the infinite hat game, but then sold it to *The Who* who changed the lyrics.

My lyrics are on the slides.

Their lyrics are in this youtube video of the Who singing it:

<https://www.youtube.com/watch?v=Fwt1adCTnd4>

Listen to their version and remember my version as sung by *Bad Sequence*.

Whose version do you prefer?

(Consider: My version is about the infinite hat game. Their version is sung by a guy so suspicious that his girlfriend is cheating that he warns her he can see what she is up to since *I can see for miles and miles*. Their lead singer and ours (Bad Sequence) are about the same for singing. Their back up singers and instrumental are better than ours.)

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6. (0 points, but you must answer it) I confess that the idea behind the comedy routine at the end of my skit where Andrew is asking *Who did you sell it to* was not mine. I adapted a well known (or is it—that's the point of this problem) comedy routine called *Whose On First* where the players on a baseball team are words like Who (the first baseman's name is Who)

It was performed by the comedy duo Abbott and Costello.

Here it is on You Tube

<https://www.youtube.com/watch?v=5FsJe4DScDs>

- a) Listen to it. (I am NOT going to ask what's better theirs or mine, since it's MUCH better, and they were first, unlike the song *I can see for miles and . . .*)
- b) Name all of the players that are mentioned in the routine.
- c) Here is my question: Have you heard it before? I ask since it was, at one time, the most famous comedy routine in America, but I suspect it has long faded. The answers I get will either confirm or refute that suspicion.