Homework 09, Morally Due 12:30PM, Tue Apr 14 2026

1. (0 points) What is your name?

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2. (30 points) **Recall** The infinite 3-ary Can Ramsey Theorem: For all $\mathrm{COL}\binom{\mathsf{N}}{3} \rightarrow [\omega]$ there is an infinite $H \subseteq \mathsf{N}$ such that one of the following 8 cases occurs.

   (If we write $\mathrm{COL}(x, y, z)$ then $x < y < z$.)

   - A $\emptyset$-homog set, so all triples are the same color.
   - A $\{1\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff $x_1 = y_1$.
   - A $\{2\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff $x_2 = y_2$.
   - A $\{3\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff $x_3 = y_3$.
   - A $\{1, 2\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff ($x_1 = y_1$ and $x_2 = y_2$).
   - A $\{1, 3\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff ($x_1 = y_1$ and $x_3 = y_3$).
   - A $\{2, 3\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff ($x_2 = y_2$ and $x_3 = y_3$).
   - A $\{1, 2, 3\}$-homog set, so $\mathrm{COL}(x_1, x_2, x_3) = \mathrm{COL}(y_1, y_2, y_3)$ iff ($x_1 = y_1$ and $x_2 = y_2$ and $x_3 = y_3$) (this is rainbow).

   If B is a set of three points in the plane then $\mathrm{AREA}(B)$ is the area of the triangle formed by the three points in B.

   Let X be an infinite set of points in the plane, no three colinear.

   Show that there is an infinite subset H of X such that the map

   $B \in \binom{H}{3}$ goes to $\mathrm{AREA}(B)$ is injective

   (So all triangles have a different area.)

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3. (30 points) Let $n \in \mathsf{N}$ and $n \geq 3$. Find the constant $\alpha, \beta \in \mathsf{Q}$ such that the following is true, and prove it:

   If $n \equiv 0 \pmod 2$ and COL: $[n]$ then there are at least $\frac{n^3}{24} + \alpha n + \beta$ mono K_3's.

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4. (40 points) In this problem we guide you through another proof of the Happy ending theorem.

For all $k \geq 3$, there exists an n, such that, for set X of n points, in the plane, NO THREE COLINEAR, there exists $Y \subseteq X$ with $|Y| = k$, and the points in Y are the vertices of a convex hull of size k.

We show that $n = R_3(k)$ suffices (for all COL: $\binom{[n]}{3} \rightarrow [2]$ there is a homog set of size k).

Let $X = \{p_1, \ldots, p_n\}$.

Let COL: $\binom{[n]}{3}$ be defined as follows:

COL$(i < j < k) =$

- RED if p_i-p_j-p_k is CLOCKWISE
- BLUE if p_i-p_j-p_k is COUNTER-CLOCKWISE

(Do you young folk even know what CLOCKWISE means, with your fancy digital watches?)

FINISH THE PROOF.