

**Homework 10, Morally Due 12:30PM, Tue Apr 21 2026**

1. (0 points) What is your name?

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2. (35 points) For this problem assume  $R(5) = 43$  (this is widely believed). Hence the following is true:

For all COL:  $\binom{[43]}{2} \rightarrow [2]$ , there exists at least ONE mono  $K_5$ .

- (a) (20 points) Fill in the following sentence and prove it:

Let  $L \in \mathbf{N}$ . Think of  $L$  as SMALL (like  $L = 2, 3, 4, 5$ .) If  $n$  satisfies condition  $XXX(L)$  then, for all COL:  $\binom{[n]}{2} \rightarrow [2]$ , there exists at least  $L$  mono  $K_5$ 's. This should use a similar technique to what I did in class with  $K_4$ . (NOTE: DO NOT do the following: for  $L = 2$ , use  $n = 2 \times 43$ , for  $L = 3$ , use  $n = 3 \times 43$ .)

- (b) (0 point and don't hand in but this will help you with Part c) Write a program that will, given  $L$ , find the least  $n$  that satisfies  $XXX(L)$ .
- (c) (15 points) Run the program for  $L = 1$  to 10 and give us a table of the following form (I only give the first three rows

$L$	least n
1	43
2	50
3	57

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3. (35 points)

**Def** Let  $X$  be a finite set. Let  $\text{COL}: X \rightarrow \omega$ . Let  $Y \subseteq X$ .  $Y$  is *homog* if all the elements of  $Y$  are the same color.  $Y$  is *rainbow* if all the elements of  $Y$  are different colors.

The following is the 1-dim Large Can Ramsey:

*For all  $k$  there exists  $n = LCR(k)$  such that,*

*for all  $\text{COL}: \{k, k+1, \dots, k+n\} \rightarrow \omega$  either there exists a large homog set or a large rainbow set.*

- (a) (35 points) Prove the 1-dim large can Ramsey theorem is true using
- (b) (0 points but you must do) Does your prove give a bound on  $LCR(k)$  (perhaps in terms of large Ramsey numbers of a higher arity)?

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4. (30 points) Let  $a, b, c \geq 2$ . Let  $R(a, b, c)$  be the least  $n$  such that, for all  $\text{COL}: \binom{[n]}{2} \rightarrow [3]$  there exists either a RED  $K_a$  or a BLUE  $K_b$  or a GREEN  $K_c$ . (So  $R(k, k, k)$  is the Ramsey Number for 3-coloring a graph and looking seeking a homog set of size  $k$ .)

(a) (0 points but you should do it) Show that

$$R(2, b, c) = R(b, c)$$

$$R(a, 2, c) = R(a, c)$$

$$R(a, b, 2) = R(a, b).$$

(b) (20 points) Show that

$$R(a, b, c) \leq R(a-1, b, c) + R(a, b-1, c) + R(a, b, c-1) - 1.$$

(c) (0 points) Write a program that will, on input  $a, b$ , outputs an upper bound on  $R(a, b)$ . Use the recurrence-inequality we had for  $R(a, b)$  and also the trick where if the bounds on  $R(a, b-1)$  and  $R(a-1, b)$  are even then the bound on  $R(a, b)$  is  $R(a-1, b) + R(a, b-1) - 1$ . (We will be running it on small enough values that doing it by recursion will be fine.)

(d) (0 points) Write a program that will, on input  $a, b, c$ , outputs an upper bound on  $R(a, b, c)$ . (We will be running it on small enough values that doing it by recursion will be fine. This program will use the  $R(a, b)$  program.) (I do not think there is a trick to lower the number by 1.)

(e) (10 points) Use your program to find upper bounds on  $R(a, b, c)$  for  $2 \leq a, b, c \leq 6$ . DO NOT report all of these values. Since Megan ONLY cares about the symmetric case, JUST output the bounds on  $R(2, 2, 2)$ ,  $R(3, 3, 3)$ ,  $R(4, 4, 4)$ ,  $R(5, 5, 5)$  and  $R(6, 6, 6)$ .

So I want a table like the one below (the numbers are made up)

$k$	bound on $R(k, k, k)$
2	7
3	100
4	1000
5	2000
6	3000